

Preface

This book provides an introduction to a newly developed bifurcation theory and its applications to dynamical systems and partial differential equations (PDEs) from science and engineering.

The first two chapters of the book contain a brief introduction to the standard bifurcation theory for nonlinear PDEs. The treatment of the classical theorems is unified by the Lyapunov-Schmidt reduction and the center manifold reduction procedures.

The next four chapters introduce a new bifurcation theory developed recently by the authors. This theory is centered at a new notion of bifurcation, called attractor bifurcation for nonlinear evolution equations. The main ingredients of the theory include a) the attractor bifurcation theory, b) steady state bifurcation for a class of nonlinear problems with even order nondegenerate nonlinearities, regardless of the multiplicity of the eigenvalues, and c) new strategies for the Lyapunov-Schmidt reduction and the center manifold reduction procedures.

With the bifurcation theory, many long standing bifurcation problems in science and engineering are becoming accessible, and are treated in the last four chapters of the book. In particular, applications are made for variety of PDEs from science and engineering, including, in particular, the Kuramoto-Sivashinsky equation, the Cahn-Hilliard equation, the Ginzburg-Landau equation, Reaction-Diffusion equations in Biology and Chemistry, the Bénard convection problem, and the Taylor problem. The applications provide, on the one hand, general recipes for other applications of the theory addressed in this book, and on the other hand, full classifications of the bifurcated attractor and the global attractor as the control parameters cross certain critical values, dictated usually by the eigenvalues of the linearized problems. It is hoped that the book will be helpful in advancing the study

of nonlinear dynamics for problems in science and engineering.

We would like to acknowledge explicitly the great impact on our career of Professors Wenyuan Chen, Jacques-Louis Lions, Louis Nirenberg and Roger Temam. We have greatly benefited from discussions with Hari Bercović, Jerry Bona, Zhimin Chen, Ciprian Foias, Michael Ghil, Michael Jolly, Benoit Perthame, Roger Temam, Xiaoming Wang, and Kevin Zumbrun, whom we warmly thank. Special thanks are due to three students, Willie Hsia, Jungho Park, and Masoud Yari, at Indiana University, who read earlier drafts of the book and made numerous helpful corrections and suggestions. The first draft was patiently typed by Ginny Jones and Shan Ma. Also we are grateful to Wen Masters and Reza Malek-Madani of Office of Naval Research for their constant support and encouragement. The final touch of the book was done while the second author was visiting DMA (Département de mathématiques et applications) and Département Terre-Atmosphère-Océan (TAO) at École Normale Supérieure (ENS) Paris; their hospitality and support are acknowledged. Also, we wish to thank our editors, Qin Jing and Rok Ting Tan, at World Scientific for their support and patience.

The research presented in this book was supported in part by grants from the Office of Naval Research, and from the National Science Foundation.

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Bloomington, Indiana
April, 2005