Preface

This monograph is devoted to the development and application of Melnikov-type methods applied to high dimensional dynamical systems governed by ordinary differential equations. There is no doubt that the analytical approaches are more powerful than numerical ones, but the key problem regarding a study of nonlinear dynamical systems concerns a narrow band of examples taken from both real live as well as engineering systems dynamics which can be studied directly in a purely analytical way. This observation is clearly exhibited by an attempt to predict chaotic behaviour of high dimensional dynamical systems. Although the classical Melnikov's technique have found various applications to predict homoclinic intersections, it is devoted only to the analysis of three dimensional systems (in the case of mechanics, they represent one-degree-of-freedom non-autonomous systems).

This monograph contributes to nonlinear dynamical systems development mainly via two features. First, we show and illustrate a way to extend the classical Melnikov's approach to study high dimensional dynamical systems. Second, our considerations include simple models of dry friction and hence, both stick-slip and slip-slip chaotic orbits occurrence can be analytically predicted. It should be noted that this part of research is very rarely reported in the existing bibliography even regarding one-degree-of-freedom non-autonomous dynamics.

In Chapter 1 a role of the Melnikov-type methods in applied sciences (applied physics, mathematics and engineering) is described. The state-of-art of the original Melnikov and the so-called Melnikov-type approaches in various branches of sciences with emphasis on their advantages and drawbacks are illustrated and discussed. In addition, special attention is paid to the Melnikov (Melnikov-type) methods devoted to the analysis of both lumped and continuous mechanical, civil engineering and electrical systems.

The critical review of the existing results in the field of classical Melnikov method application and beyond allows to discover a gap and limitation in the so-far applied techniques to predict homo- and heteroclinic bifurcation yielding chaotic orbits.

First, it has been strongly emphasized that the Melnikov's method belongs to the perturbational (asymptotic) techniques , and hence it is opened for further development and modification within the field of applied mathematics.

Second, the critical overview of the research devoted to either application or extension of the Melnikov technique indicates a lack of modification in the method towards a study of higher order dynamical systems, i.e. systems with a few degrees-of-freedom in the language of mechanics.

The two mentioned important observations motivate the authors to focus on the problems of developing the existing classical Melnikov approach to analyse multibody mechanical systems.

Another important feature of the book is a hybrid and unified approach which draws on both smooth and nonsmooth (friction) dynamical systems.

In Chapter 2 the classical Melnikov approach regarding the analysis of smooth systems is revisited. This chapter plays an introductory role, and is slightly modified with respect to the original Melnikov approaches in order to fit suitably with the rest of the book material. Splitting of the homoclinic orbit that occurred in a studied three dimensional dynamical system is illustrated and rigorously discussed yielding the Melnikov function used as a criterion of chaotic threshold occurrence.

In Chapter 3, a Froude pendulum harmonically driven and with dry friction is studied. Since the studied system possesses one-degree-of-freedom, the classical Melnikov approach is applied to derive Melnikov function governing chaotic dynamics occurrence. However, in contrary to the standard application of the Melnikov method, our studied systems exhibit (for some parameters) two new homoclinic orbits giving additional analytical thresholds of stable and unstable homoclinic manifold intersections. In addition, we show how to deal with Melnikov integrals to include a role of Coulomb type friction on the system dynamics, i.e. to derive analytical formulas for the Melnikov function including both smooth and non-smooth dynamic chaos.

The obtained analytical thresholds of smooth/nonsmooth potential chaotic dynamics have been verified numerically to show a good match between the theoretical analysis and the introduced mathematical model governing the dynamics of the planar Froude pendulum as non-autonomous dynamics with dry friction.

In Chapter 4 a more precise definition of the domains of the stick-slip and slip-slip chaotic dynamics of a one degree-of-freedom very weakly forced oscillator using a novel approach based on the analysis of wandering trajectories is formulated. A comparison with analytical prediction obtained using Melnikov's technique has demonstrated a good agreement with the results presented. Note that all the standard numerical methods, in particular the direct computations of Lyapunov exponents, are time consuming. The presented approach is effective, convenient to use and requires much less computational time in comparison with other approaches.

Chapter 5 is devoted to the extension of the classical Melnikov technique to the multibody mechanical systems. First, modeling of lumped mechanical systems are applied in a way suitable for further application of the Melnikov-type extension. Namely, the so-called Melnikov-Gruendler method is introduced, and the asymptotical behaviour of the fundamental matrix solutions and hence of the associated Melnikov functions are studied.

The second step includes the analysis of two-degrees-of-freedom mechanical systems, and the linearization procedure along a homoclinic orbit.

It is shown, among others, that a lack of coupling of variational equations computed along a homoclinic orbit as well as their symmetry allow for an efficient derivation of the associated Melnikov functions. Finally, we show that the Melnikov-Gruendler approach is reduced to the Melnikov original method for one-degree-of-freedom systems, and hence leading to the study of high dimensional systems via the extension of Melnikov's original approach.

Chapter 6 is mainly focused on analytical prediction of chaos occurrence in a self-excited (due to Coulomb type friction) and harmonically driven spherical pendulum. First, dimensionless two differential equations are derived and then transformed to a suitable form for further application of the earlier introduced Melnikov-Gruendler approach. The obtained analytical criterions for chaos occurrence (homoclinic intersection of stable and unstable manifolds) have been verified via standard numerical tools, i.e. bifurcation diagrams and phase space projections, showing surprisingly accurate prediction.

A double self-excited Duffing-type oscillator with dry friction modeled as the third order polynomial with the relative velocity and being harmonically excited is studied in Chapter 7. The studied system is formulated in a general form, since it includes internal, aerodynamic and hydrodynamic frictions, and hence the further obtained results can be directly used by engineers/researchers dealing with the particular examples of 2-DOF oscillators. The system is transformed to the non-dimensional form and then it is further studied with the help of the Melnikov-Gruendler approach. Finally, various criterions of smooth/nonsmooth chaos are derived analytically and are then successfully verified numerically. In addition, the critical surface in three-dimensional parameter space for homoclinic intersections is constructed and the predicted occurrence of chaos is also verified numerically.

In Chapter 8 a triple self-excited (friction) Duffing-type oscillator is studied. It consists of three masses m lying on a rigid belt moving at constant velocity v. Dry friction (approximated by a modified function composed of a "sign" and a polynomial) occurs between masses and the belt. External masses are linked to a basis through elastic Duffing-type elements and viscous dampers with relatively small damping forces. Middle mass is coupled to external masses by nonlinear springs with arbitrary characteristics satisfying the condition $k_0(-z) = -k_0(z)$. One of the external masses is harmonically driven with frequency Ω and relatively small amplitude Γ. Since the studied system has three-degrees-of-freedom, the Melnikov-Gruendler technique is applied to predict homoclinic bifurcations. The various analytical criterions for homoclinic chaos occurrence are derived.

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Jan Awrejcewicz and Mariusz M. Holicke