Errata and Additions to Pricing Derivative Securities

T. W. Epps

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1. p. 27, last line before (2.7): "be a monotone sequence, and let μ be a measure on $(\Omega, \mathcal{C})$ such that $\mu(A_1) < \infty$ if $\{A_n\} \downarrow$."
2. p. 30, eqn. (2.8): $\int_a^x f(t) \cdot dt$.
3. p. 36, Theorem 2, second integral: $\int \lim_{n\to\infty} g_n(\omega) \cdot d\mu(\omega)$.
4. p. 47, line 2: page 36.
5. p. 59, last line: holds if $\mu_k^{'}$ exists.
6. p. 61, line 2: $a + bEX$
7. p. 63, lines 5-6: while the derivatives evaluated at $\zeta = 0$ are proportional to the probabilities, as $\Pi^{(k)}(0)/k! = f(k)$.
8. p. 80, fourth line from bottom: $\mathsf{P}(V = \sigma_j^2) = p^{2-j}(1-p)^{j-1}$.
9. p. 87, line -1: $X_s + b_s E_s Y_{s+1} + ... + E_s\left[b_{t-1}\left(E_{t-1}Y_t\right)\right]$
10. p. 88, Example 34: $E_s X_t = E_s(E_t Z) = E_s(Z) = X_s$.
11. p. 89, line 8: that simply....
12. p. 94, para. 2, line 4: The process thus has time-stationary and independent increments.
13. p. 97, line 1: as $n \to \infty$.
14. p. 104 property #1 of the integral: $\lim_{n\to\infty} \left| b_t(W_{t+1/n} - W_t) \right| = 0$.
15. p. 108, para. 3, lines 1-2: quadratic variation of Brownian motion itself is....
16. p. 109. 1st line should be: Approximating $X_{t_j} - X_{t_{j-1}} = \int_{t_{j-1}}^{t_j} a_s \cdot ds + \int_{t_{j-1}}^{t_j} b_s \cdot dW_s$ as $a_{t_{j-1}}(t_j - t_{j-1}) + b_{t_{j-1}}(W_{t_j} - W_{t_{j-1}})$, the quadratic variation of $\{X_t\}$ on the grid $\{t_j\}$ is approximately
17. p. 109, line 4: Again setting $t_j = jT/n$,....

18. p. 113, second limit: $\sum_{j=1}^{n} f_{XX}(X_{t_j^*})a_{t_{j-1}}b_{t_{j-1}}(t_j - t_{j-1})(W_{t_j} - W_{t_{j-1}})$:

$$\begin{aligned} &\leq \sqrt{tn^{-1}\sum_{j=1}^{n} f_{XX}(X_{t_j^*})^2 a_{t_{j-1}}^2 b_{t_{j-1}}^2 (t_j - t_{j-1}) \sum_{j=1}^{n} (W_{t_j} - W_{t_{j-1}})^2} \\ &= \sqrt{tn^{-1}\sum_{j=1}^{n} f_{XX}(X_{t_j^*})^2 a_{t_{j-1}}^2 b_{t_{j-1}}^2 (t_j - t_{j-1}) \cdot tn^{-1}\sum_{j=1}^{n} Z_j^2} \\ &\to 0, \end{aligned}$$

19. p. 115, Example 49, line 8: $W_t \cdot dW_t = dW_t^2/2 - dt/2$.

20. p. 159, line 2 of text: Let....

21. p. 160, para. 2, line 8: so that the normalized value....

22. p. 161, last line of text: and $p, c \in \Re$.

23. p. 164, line 7: underlying is a futures price or a no-dividend stock....

24. p. 168, line 3: $S_t + K$ worth of stock and invest the dividends in stock as received.

25. p. 168, line -11: $S_t e^{-\delta(T-t)}$ worth of stock, financing the dividends by selling more.

26. p. 170, col. 3 line 1 of table: $C^E(S_t, T-t) = S_t + K$.

27. p. 171, line 4 of para. headed "Variation with Term": ...plus a T_2-expiring call that kicks in at T_1 if the first call was not exercised.

28. p. 175, FALSE Theorem, line 1: $S_t \geq 0$.

29. p. 194 line 5, add after "risk-neutral" pricing: when the money fund is numeraire.

30. p. 194 lines 9-10: as well as an "equivalent-martingale" measure.

31. p. 196, line 13: ...violating both conditions.

32. p. 197, add just before section “Complete Markets”: However, one sees immediately that the measure $\widetilde{\mathsf{P}}$ is not a risk-neutral measure, in that $s_0 \neq b(0,n)\widetilde{E}s_n$.

33. p. 198 change first two expressions to

$$\hat{v}_0(A_j) \equiv s_{00} \cdot \hat{E}\left(1_{A_j}/s_{0j}\right) = s_{00} \int_{A_j} s_{0j}(\omega)^{-1} \cdot d\widehat{\mathsf{P}}(\omega)$$

or

$$\tilde{v}_0(A_j) \equiv s_{00} \cdot \tilde{E}\left(1_{A_j}/s_{0j}\right) = s_{00} \int_{A_j} s_{0j}(\omega)^{-1} \cdot d\widetilde{\mathsf{P}}(\omega).$$

34. p. 213, last line: $P^A(s_0, n) = X - s_0$.

35. p. 219, line -3: From k onward ....

36. p. 220, replace second line as: At $k-1$, the critical stage just before the ex date, one compares the derivative’s current exercise value, which is based on the cum-dividend price, with its value if held until the stock price reacts to the dividend: .

37. p. 220, expression at line 3 should be

$$D_{k-1}(s_{k-1}) = \frac{1}{1+r}\hat{E}_{k-1}D_k[s_k^c(1-\delta)] \vee D_{k-1}^X(s_{k-1}^c).$$

38. p. 222, 2 lines above expression, replace “thereafter” with “from k onward”.

39. p. 223, 2d. line after (5.40): $R_{j+1} \in \{d, u\}$.

40. p. 225, replace first 4 lines with: ... and off the ex-dividend price from k onward, as

$$D_j(s_j) = \frac{1}{1+r}\hat{E}_j D_{j+1}(s_{j+1}) \vee D_j^X(s_j);$$

while at $k-1$

$$D_{k-1}(s_{k-1}) = \frac{1}{1+r}\hat{E}_j D_k(s_k^c - \Delta) \vee D_{k-1}^X(s_{k-1}^c).$$

41. p. 228, line 9: Since $e^{\bar{r}_j T/n}$ corresponds to $(1+r_j)$....

42. p. 231 last 2 lines of 2d para: so that $\hat{E}_0 S_T = S_0 e^{r(0,T)} = S_0 B(0,T)^{-1}$ and $S_0 = B(0,T)\hat{E}_0 S_T$.

43. p. 247, 6th line from bottom: section 5.6.1.

44. p. 261, line 2: first expression should be $D(S_t, T-t) = p_t S_t^c + q_t M_t$

45. p. 264, line 7: page 117.

46. p. 265, add after 1st para: Just as was seen to be true in the Bernoulli setting, a different measure is required to turn normalized prices into martingales if S_t^c replaces M_t as numeraire. Letting $M_t^{**} = M_t / S_t^c$ and applying Itô gives
$$dM_t^{**} = (r_t - \delta_t + \sigma_t^2 - \mu_t) \cdot dt - \sigma_t \cdot dW_t.$$
With
$$Q_T' = \exp\left[\int_0^T \sigma_t^{-1}(r_t - \delta_t + \sigma_t^2 - \mu_t) \cdot dW_t - \frac{1}{2}\int_0^T \sigma_t^{-2}(r_t - \delta_t + \sigma_t^2 - \mu_t)^2 \cdot dt\right]$$
and
$$\widetilde{\mathsf{P}}(A) = E\mathbf{1}_A Q_T' = \int_A Q_T'(\omega) \cdot d\mathsf{P}(\omega),\ A \in \mathcal{F},$$
we have a measure under which $\widetilde{W}_t = W_t - \int_0^t \sigma_u^{-1}(r_u - \delta_u + \sigma_u^2 - \mu_u) \cdot du$ is a Brownian motion and M_t^{**} becomes trendless, with $dM_t^{**} = -\sigma_t \cdot d\widetilde{W}_t$. However, although $\widetilde{\mathsf{P}}$ does make $\{M_t^{**}\}$ a martingale, it is no longer a "risk-neutral" measure; for example,
$$\widetilde{E}_t S_T^c = S_t^c \exp \int_t^T (r_u + \sigma_u^2) \cdot du$$

47. p. 265, line -9: represented as an Itô process...

48. p. 267, first line after (6.15): where $\hat{Z} \sim N(0,1)$.

49. pp. 267-8, replace entire section headed "Futures Prices under the Martingale Measure" as:

    Modeling the evolution of a futures price over some interval $[0,T]$ as geometric Brownian motion under the canonical measure P,
$$d\mathsf{F}_t = \mu_t \mathsf{F}_t \cdot dt + \sigma_t \mathsf{F}_t \cdot dW_t, \tag{1}$$

let us consider the implications for the stochastic behavior of F_t, the value of a futures *position* that was initiated at $t = 0$. Supposing this to have been marked to market at times $0 < t_1 < ... < t_n \le t$, with gains (losses) being invested in (financed by) shares of the money fund, we have

$$\mathsf{F}_t = (\mathsf{F}_{t_1} - \mathsf{F}_0)\frac{M_t}{M_{t_1}} + (\mathsf{F}_{t_2} - \mathsf{F}_{t_1})\frac{M_t}{M_{t_2}} + ... + (\mathsf{F}_{t_n} - \mathsf{F}_{t_{n-1}})\frac{M_t}{M_{t_n}} + (\mathsf{F}_t - \mathsf{F}_{t_n}).$$

Now, as an approximation, imagine that the marking to market has taken place *continuously*. Taking limits in the above as $\max(t_j - t_{j-1}) \to 0$ gives

$$\mathsf{F}_t \doteq M_t \int_0^t M_s^{-1} \cdot d\mathsf{F}_s$$

and, upon normalizing by dividing by M_t,

$$d\mathsf{F}_t^* = d(\mathsf{F}_t/M_t) = M_t^{-1} \cdot d\mathsf{F}_t = \mu_t \mathsf{F}_t^* \cdot dt + \sigma_t \mathsf{F}_t^* \cdot dW_t.$$

Since it is the futures *position* rather than the futures *price* that has the status of an asset, it is F_t^* rather than F_t^* that should be a martingale under appropriate change of measure. Applying Girsanov's theorem with

$$Q_T = \exp\left[-\int_0^T \sigma_t^{-1}\mu_t \cdot dW_t - \frac{1}{2}\int_0^T \sigma_t^{-2}\mu_t^2 \cdot dt\right]$$

produces the measure $\widehat{\mathsf{P}}$ that makes $\widehat{W}_t = W_t + \int_0^t \sigma_u^{-1}\mu_u \cdot du$ a Brownian motion and thereby removes the trend in F_t^*, as $d\mathsf{F}_t^* = \sigma_t \mathsf{F}_t^* \cdot d\widehat{W}_t$. Referring to (1), we see that under this measure

$$d\mathsf{F}_t = \sigma_t \mathsf{F}_t \cdot d\widehat{W}_t,$$

so that the *un*normalized futures price is itself a trendless geometric Brownian motion, and hence a martingale. Accordingly, the time-T futures price has the following distribution conditional on what is known at $t \le T$:

$$\mathsf{F}_T \sim \mathsf{F}_t \exp\left(-\frac{1}{2}\int_t^T \sigma_u^2 \cdot du + Z\sqrt{\int_t^T \sigma_u^2 \cdot du}\right), Z \sim N(0,1).$$

With this result valuing European options or other European-style derivatives on the futures is again reduced to finding the discounted expectation of a function of a lognormally distributed random variable.

50. p. 269, 7th line from bottom: page 153.

51. p. 302, caption for fig 7.1: replace δ with Δ.

52. p. 312, line 3: infinite-horizon version of (7.4).

53. p. 386, last line: European call is then $C^E(S_t, T-t) = B(t,T)\hat{E}_t(S_t - X)^+$, where the expectation is given by....

54. p. 397, line 7: ...that solve (8.19) and (8.20) also solve (8.18)

55. p. 400, change first sentence as:

    Therefore, the current value of the option under stochastic volatility is simply the mathematical expectation of $P^E(S_0, T; \bar{\sigma}_T)$, or

    $$P^E(S_0, T) = \hat{E}P^E(S_0, T; \bar{\sigma}_T).$$

56. p. 401, lines 2-3: evaluating $\bar{\sigma}_T$ for each path, and averaging $P^E(S_0, T; \bar{\sigma}_T)$ over the replications.

57. p. 404, line 9: $\mathbf{1}_{(-\infty,x)}(s_T)$

58. p. 420, 4th line after (9.10): $d\ln S_t = (\alpha - \theta\upsilon - \sigma^2/2)\cdot dt + \sigma \cdot dW_t + (\ln S_t - \ln S_{t-})$.

59. p. 420, 7th line after (9.10): Replace two lines of expression with

    $$\ln S_T - \ln S_t = (\alpha - \theta\upsilon - \sigma^2/2)(T-t) + \sigma\,(W_T - W_t) + \sum_{j=0}^{N_T - N_t} \ln(1 + U_j)$$

    with $U_0 \equiv 0$.

60. p. 420, lines -7,-8: A convenient assumption is that $\{1 + U_j\}_{j=1}^{\infty}$ are i.i.d. as lognormal, ...

61. p. 434, line 11: $N\left[\mu, \sigma^2(\nu - 1)/U\right]$.

62. p. 438, line 2:

    $$f_{R_{0,t}}(R) = \frac{2e^{\gamma(R-\mu t)/\sigma^2}}{\nu^{t/\nu}\sqrt{2\pi\sigma^2}\,\Gamma(t/\nu)}\left(\frac{|R-\mu t|}{\rho}\right)^{\frac{t}{\nu}-\frac{1}{2}} K_{\frac{t}{\nu}-\frac{1}{2}}\left(\rho\frac{|R-\mu t|}{\sigma^2}\right)$$

63. p. 516, line 7:

$$d_s^{\pm}(s,t) = \frac{\pm d(s \pm \Delta s, t) \mp d(s,t)}{\Delta s}$$

64. p. 518, line -8:

$$+\frac{d(s+\Delta s, t+\Delta t) + d(s-\Delta s, t+\Delta t) - 2d(s, t+\Delta t)}{\Delta s^2} \cdot q.$$

65. p. 524, equation (12.15):

$$l_i = b - a \cdot \frac{c}{l_{i-1}}$$

66. p. 524, line -2: $k_1 = e_1 - a \cdot d_0$.

67. p. 666, reference [64]: Bernoulli 1, 281-299.

68. p. 668, reference [88]: pub. date (1979).