

Book Report: **The Kelly Capital Growth Investment Criterion**, by Leonard MacLean, Edward Thorp, and William Ziemba (editors).

This book provides a comprehensive survey of research and applications on a special utility function – the log-utility, known therein as the growth-optimal, or Kelly strategy (for one of the early proponents). What has caused such interest in this utility function?

Compelling action cinema: A mild mannered mathematics professor stumbles upon a “secret” formula which allows him to turn the tables on the casinos by developing a card counting system for blackjack which can generate expected PROFITS over time. But the casinos fight back. They institute rules to prevent our hero from participating in his plans. The hero tries various ways to disguise his looks, but to little avail. Eventually, the hero turns to other areas to become rich by applying his skills to equity markets (for example in evaluating mispriced options). There is even a side-story that our hero reportedly discovered the Black-Scholes option pricing formula before B/S, but did not publish it. And the happy ending: The hero becomes rich by applying the theory to the investment domain. (But he is eventually is knocked down by an overzealous federal prosecutor).

The secret formula is the topic of this book: the log-utility function. Many of the properties of the Kelly strategy have been understood for a long time. The approach is a special case of general expected utility maximization. The basics of decision making under uncertainty via the von Neumann approach are readily known to the reader. Given five straightforward assumptions, the decision maker -person or organization- DM will choose that alternative which provides the highest certainty equivalent (CE) value. A short-cut is to compute the expected utility EU of the outcomes for each alternative and again chose the one with the highest expected utility (highest CE = $U^{-1}(EU)$). It is assumed that the probabilities of the outcomes and the consequences of the outcomes are given at the time of the decision. What can be simpler?

Unfortunately, over the past sixty years, there have been endless debates/studies/arguments- about both the theoretical and practical considerations regarding the EU approach. For example, behaviorists have shown persuasively that humans do not apply EU when left to their own affairs –even experts in a given field. Humans are inconsistent decision makers – risk averse in some circumstances and risk seeking in others, for instance. Also, individuals do not discount utility over time in a manner consistent with the standard discount formulae. Another line of critique centers on missing elements in the theory – such as the need for principles, goals, and ethical aspects; see Robert Nozick, **The Nature of Rationality** (Princeton University Press, 1993) and its extensive references.

Given this situation, it is heartening to discover an extensive book (853 pages) devoted to the topic of employing one particular utility function with desirable properties. Anyone who has taught or applied a careful decision analysis realizes that the selection of an acceptable utility function presents one of the

most challenging elements for a successful application. When introduced to the topic, many students find the decision tree structure compelling for smaller repeating decisions for which risk neutrality is appropriate. Once risk aversion is introduced, there is much ambiguity about the most desirable utility function. In conjunction, the temporal issues can be confusing and do not correspond to intuition for many people; discounting expected utility avoids the important issues of achieving target goals, for example, and hyperbolic considerations.

As the book discusses throughout, there are numerous advantages to the Kelly strategy, especially for long-term investors who face a long sequence of similar decisions. Briefly, this strategy possesses highly desirable convergence properties as the number of decisions increases to infinity. It outdistances other utility functions, for example. Also, the function satisfies the standard criteria of risk aversion and decreasing marginal risk aversion – thus leading to consistent decision making over time. These appropriate properties are important for normative studies.

The structure of the book is as follows. The book presents a compendium of articles covering the theory and practice regarding the log-utility function -- 54 chapters in total divided into six parts – 1) early ideas; 2) classic papers; 3) asset allocation; 4) good and bad properties; 5) utility foundations; and 6) evidence of investors' use of the Kelly strategy. The majority of chapters are reprints from a variety of publications – 40 out of 54. Each of the six parts is briefly summarized in turn by the editors. Eight chapters are newly written papers (including one by this reviewer). The primary application areas involve financial investments and games of chance such as blackjack and horse race bets.

Pros of log-utility: The significant properties of the Kelly strategy are summarized in Part 4, as well as elaborated in many other sections of the book. A few of the pros are: the long-term convergence of the investor's capital is maximized by following the Kelly strategy when the random variables are IID and transaction (market impact) costs can be ignored – thus the term growth-optimal utility function. The Kelly strategy maximizes the expected probability of achieving a target wealth (again a convergence result). Under continuous time processes, there is no chance of bankruptcy: $\ln(0)$ is undefined. Noteworthy investors and academics have touted the Kelly strategy for a number of applications. The most famous is one of the editors, Edward Thorp, who showed that individuals can gain an edge in Blackjack by counting schemes (**Beat the Dealer**, Vintage Books Edition 1966). Ed has been a tireless advocate for applying the Kelly strategy.

Cons of log-utility: On the other side, there have been a number of critiques of the Kelly strategy. The most significant and critical was by Paul Samuelson; three of his papers are reprinted in the book. Much of the difficulty centers on the behavior of Kelly under a finite set of decisions, and especially for a small-to-modest number. The great majority of investors are unconcerned about the limit of an infinite series. As mentioned in the book, an investor who follows the Kelly strategy can be subjected to severe downside consequences over a small or even a moderate number of decisions.

Importantly, the tail risks over a finite sequence of decisions are not readily addressed by expected utility maximization. The effort by many financial experts on measuring and controlling worst-case outcomes, such as Conditional-Value-at-Risk, are largely ignored. In addition, when the decisions are

made by levered financial institutions and individuals over a finite number of time steps, the possibilities of bankruptcy, no matter how remote, must be avoided (since the log of non-positive numbers is undefined). Thus, applications such as re-insurance of very low probability, high severity events would be difficult under the Kelly strategy.

The log-utility function presents a special case of the iso-elastic function – $U(w) = (1/\gamma) * w^{\gamma}$, where γ is the sole risk aversion parameter. The log-utility is equivalent to $\gamma = 0$. This family of functions is simple and has positive characteristics. However, temporal and goal oriented issues are difficult to model without additional constraints or rules. For example, the investor may wish to reach a retirement goal at age 65. Her investment strategy can be tailored to individual circumstances. For example, once the target wealth is reached before age 65, the investor may switch to a riskless portfolio. Dynamic strategies of this type are mostly inaccessible by iso-elastic utility functions alone.

Another area of concern involves misspecification of stochastic parameters. In many cases, the investor may decide to implement a developed policy rule. However, upon application, the investor finds that its performance is less than anticipated. It is difficult to distinguish if the poor performance is due to unlucky circumstances or incorrect parameters when large drawdown periods are possible (but not likely).

Alternatives to log-utility: To overcome these difficulties, a number of alternatives are discussed in the book. To the point of large drawdowns, the investor may wish to implement fractional Kelly (bet a fixed proportion of Kelly at each point such as one-half Kelly). These strategies reduce the worst-case consequences, but can be difficult to implement when many correlated securities are present. An alternative is to apply a more conservative iso-elastic function such as $\gamma = -1$.

Another approach is to couple Kelly with drawdown constraints. This reviewer's article in the book discusses this topic with several examples. Similarly, one could apply Kelly within a stochastic control or stochastic programming context, in which the utility parameters are modified under time varying conditions – such as the time to the horizon and current investor wealth. In a similar fashion, modern risk functions other than drawdown can be integrated within a more general setting.

Legendary investors: Part 6 of the book takes up the topic of legendary investors such as Warren Buffet and George Soros. The editors suggest that these investors display Kelly-like behavior in their decisions, with some evidence. Mostly, since these investors have displayed such long-term superior performance, they must have been employing a decision process in a Kelly-like manner. Unfortunately, in most cases, except for Ed Thorp, we do not have direct evidence that the Kelly strategy factors into the superstars' decisions in a meaningful manner. It would seem to be impossible to infer the underlying utility function for any single sequence of outcomes without knowing the alternative choices, the probabilities of the outcomes at each stage, and other relevant circumstances. Too bad that we can't simply find Warren Buffett's secret formula and apply it ourselves.

In summary, the Kelly strategy has had a fascinating history and has been the subject of a large number of authors and investors. In the book's chapters, the underlying properties of this utility function are extensively developed and presented. Any investor with an eye to improving their long-term

performance should be aware of the Kelly strategy and its cousins. This book provides a fine coverage of these topics from original sources and recent research publications. The editors should be commended for their efforts to help improve decision making and thereby increase investors' capital over time.