1.4 Theory and practice

By its very nature, largely due to its connection to human behavior, financial market seldom exhibit rationality, regularity and predictability. Mathematical finance is at best an effort to get as much as possible rationality out of such a situation.

We conclude this chapter by mentioning one positive feature that enables mathematical models to work: the concept of arbitrage-free, and one negative factor that makes whatever effort hopeless: market crashes.

Arbitrage-free market

The single most important assumption that enables pricing with mathematical models possible is the idea of *arbitrage-free* or the *absence of arbitrage.*

By *arbitrage* we mean the possibility of obtaining profit without any risk. This is also commonly referred to as "free lunch." In an ideal market, such possibility should not exist, for otherwise such possibility leads to violation of "conservation wealth." For example, if all investor uses such possibility, then everybody gets rich with no suffering.

In reality, arbitrage possibility may exist for a very short time before equilibrium takes effect and eliminates such possibility. More realistically, the problem is an subjective one: one needs to formulate *risk* precisely in order to judge whether there is arbitrage opportunity. For example, in every financial model, the variance of a stock and the so-called "riskless interest rate" are at best a guess work and hence the difficulty of translating the apparent discrepancy into actual arbitrage.

Nevertheless, the fundamental assumption behind almost all mathematical models of finance, in one form or another, is the assumption of *absence of arbitrage*, *i.e.* financial claims follow the *arbitrage-free* behavior.

When market crashes

One thing everyone knows for sure about any market is that it did and it will crash from time to time. Moreover, crash strikes with even less forewarning than big earthquakes.

One most recent example is the crash following the terrorist attacks of September 11, 2001, although the market did almost fully recover not long after the event.

Other major examples include the October Crash of 1987, the Asian Crisis of 1997 and the Russian Default of 1998.

When a crash occur, parameters in a mathematical model would be significantly altered to make it unusable.

However, in all the above instances, market do recover and stabilize within six months or so, and new model will be needed again.

Some reading material

The following is a list of recommended reading for further study of mathematical finance: [Avellaneda, 1999], [Baxter and Rennie, 1996],[Campbell et al., 1996], [Chriss, 1997], [Clewlow and Strickland, 1998], [Davis et al., 1995], [Dempster and Pliska, 1997], [Duffie, 1996], [Franses and Dijk, 2000], [Hull, 1993], [Jarrow, 1995], [Jarrow and Turnbull, 1994], [Lipton, 2001], [Lo and MacKinlay, 1999], [Malkiel, 1973], [Mikosch, 1999], [Musiela and Rutkowski, 1998], [Neftci, 1996], [Pliska, 1997], [Shaw, 1998], [Shiryayev, 1999], [Tavella et al., 2000], [Zhang, 1998].