Preface

As we have just entered into the new millennium, two unstoppable processes are taking place in the world:

- the globalization of the economy;
- the information revolution.

As a consequence, there is greater participation of the world population in capital market investment, such as bonds and stocks and their derivatives: options, contracts, swaps, ... *etc.* Therefore there is need for risk management and analytic theory explaining the market. This leads to quantitative tools based on mathematical methods, *i.e.* the theory of mathematical finance.

Ever since the pioneering work of Black, Scholes and Merton in the 70's, there is rapid growth of the study of mathematical finance, with involvement of ever-sophisticated mathematics. However, from the practitioner's point of view, it is desirable to have simpler and more useful mathematical tools.

It is therefore a primary goal of this book to introduce the hypermodel method (based on Robinson's infinitesimal analysis), as a simple and practical but mathematically rigorous technique for modelling finance.

Consequently, this book is suitable for students and practitioners in financial markets who

- want to acquire fluency in applicable mathematical modelling tools
- *but* have in possession only minimum mathematical sophistication such as first year calculus.

A secondary goal of the book is to indicate to mathematicians that application of hypermodel techniques in finance is both possible and fruitful.

In mathematical finance, one often encounters the following dilemma: discrete-time models are easy to use but inadequate in applications; on the other hand, although continuous-time models are strong enough in solving problems, the theory requires mathematical sophistication which is beyond the reach of most practitioners—such as stochastic analysis. Our new approach here bypasses this difficulty by taking advantage of the so-called hyperfinite time set, which can be manipulated formally as a finite object and yet it contains all information about continuous time. Indeed, this hypermodel approach is both intuitive for practical needs and rigorous in mathematical details.

It is clear that the full potential of hypermodels in mathematical finance is far from being materialized by a volume such as this, as it is probably the first one of this kind and as the author's ability is limited. Nevertheless the purpose of this book is considered fulfilled if further research in this area can be stimulated by its appearance.

The book is organized as follows.

In Chapter 1, we introduce the basic concepts, definitions and practice in financial markets. The main objects of study of mathematical finance are defined, namely stocks, bonds and options.

In Chapter 2, we outline the mathematics that we need and explain hypermodel approaches to modelling.

In Chapter 3, the absence of arbitrage, the main assumption behind mathematical finance, is developed through the use of hypermodels, from it the Black-Scholes type equations for options can be derived.

In Chapter 4, explicit solution to European option pricing and that of European barrier option are derived.

In Chapter 5, the binary tree hypermodel approach, based on the Cox-Ross-Rubinstein theory is developed, and pricing formulas are derived. Numerical examples for various options are given.

In Chapter 6, further applications to the Greeks and the term structure of interest rates are given. Of particular interest is the simple derivation of the Malliavin weight formula from the hypermodel.

Chapter 7 is an independent unit that gives more complete coverage of the mathematics behind hypermodels, *i.e.* infinitesimal analysis. This chapter is intended for readers who are mathematically matured and interested in knowing more the mathematical background.

Chapter 8 contains various kind of *MATHEMATICA* programs that practitioners may easily adapt for their own purpose.

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