tions. However, Vasicek model is not without some limitations; namely, it cannot be fitted sufficiently accurately to the market data, and it allows interest rates to become negative, i.e., it suffers from the same difficulty as the original Bachelier model for equities. While the first drawback is easy to rectify, as was done by Hull and White (1990), the second one is an inherited feature of the model and cannot be helped. In order to guarantee the positivity of interest rates one needs to use other stochastic processes with mean reversion, such as the Feller square-root process used by Cox, Ingersoll and Ross (1985), or the log-Ornstein-Uhlenbeck process used Black and Karasinski (1991). An alternative idea of dealing with fixed income derivatives is based on studying the yield curve in its entirety, as described by Heath, Jarrow and Morton (1992) and Brace, Gatarek and Musiela (1997) in continuous and discrete settings, respectively. In spite of the progress made by the above mentioned authors and many others, an adequate versatile model for pricing and hedging of fixed income derivatives is still missing.

1.3 Forex as an asset class

As was mentioned earlier, the daily turnover of forex markets is approximately $1.5 trillion. The majority of transactions with foreign exchange are executed from London and New York (about 1/3 and 1/5 of the total, respectively), as well as Tokyo, Singapore, Frankfurt, Zurich, etc. Market participants include governments, banks, international corporations, mutual and hedge funds, and individual investors. With increasing globalization of the world financial system, the role of forex as an important asset class in its own right on a par with more traditional equity and fixed income instruments becomes more and more apparent. Indeed, it is necessary to have an exposure to forex in order to be able to invest in the global markets and create a well-balanced portfolio.

When investors based in a particular country put their money in domestic bonds and equities, they are not (directly) affected by the forex rate fluctuations and all the uncertainties they have to face are domestic in nature. However, if they decide to invest their money in foreign bonds and equities, they first have to convert the domestic currency (say US dollars) into the foreign currency (say euros) at the deterministic spot rate in order to purchase foreign securities, and then, at some time in the future, they have to convert the foreign currency generated by these securities into the domestic currency (at the unknown rate prevailing at that time). Thus, investors have to deal with uncertainties both at home and overseas, and, in addition, with changes in forex rate. Nonetheless, foreign investments can generate returns which are so attractive that additional risks are worth taking.

For simplicity we consider only investments in domestic and foreign fixed income instruments. In order to quantify uncertainties associated with these investments, we need to know the prices of domestic and foreign zero coupon bonds, i.e., the price in dollars (euros) at time $t = 0$ of the obligation to pay one dollar (euro) at time T in the future. We denote the price of the domestic and foreign bonds maturing at time T by $B^0_{0,T}$ and $B^1_{0,T}$, respectively. Assuming that the bond prices are deterministic (this is a strong assumption which is made in order to simplify the exposition), we can represent the bond prices at time t as

$$B^0_{t,T} = \frac{B^0_{0,T}}{B^0_{0,t}}, \qquad B^1_{t,T} = \frac{B^1_{0,T}}{B^1_{0,t}}.$$

In addition, we need to know the forex rate S_t, i.e., the number of dollars one needs to pay at time t in exchange for one euro. The dimension of S_t is dollar/euro. In contrast to bond prices which are deterministic in our simplified framework, the forex rate is random, so that its value at some future time T is uncertain. We use the domestic bond as a benchmark for measuring the rate of return on different investments. It is clear that the relative rate of return on investment in domestic bonds is zero. The relative rate of return on investment in foreign bonds is

$$\hat{r} = \frac{B^0_{0,T} S_T}{B^1_{0,T} S_0} - 1. \tag{1.1}$$

It can be both positive and negative. Thus, in order to achieve relative rates of return above zero, domestic investors have to put some of their money overseas.

For simplicity, in this chapter we assume that

$$B^0_{t,T} = e^{-r^0(T-t)}, \qquad B^1_{t,T} = e^{-r^1(T-t)},$$

where r^0 and r^1 are constant domestic and foreign interest rates, respectively.

1.4 Spot forex

Perhaps the simplest transaction in the forex market is the exchange of two currencies at the (fluctuating) spot rate prevailing at the time of the exchange. As a rule, all exchanges are conducted through middlemen called market makers, rather than directly between two interested parties, which explains the