

Part I

**BASIC INGREDIENTS**



# Chapter 1

## Introduction

Since the 1970s, we have seen voluminous literature on the analysis of economic problems with asymmetric information. Harsanyi's (1967/68) Bayesian game and Bayesian equilibrium have served as a conceptual foundation for these analyses. While the literature provided new insights into the workings of the present-day economy which the traditional neoclassical paradigm failed to analyze, many works actually postulated a quite restrictive mode of players' interaction, that is, the principal-agent relationship (Stackelberg's leader-follower relationship), a specific instance of the non-cooperative game.

Parallel to the development of Bayesian analyses of the noncooperative game, there has been development in static descriptive cooperative game theory, the theory which analyzes another interactive mode in which several players, with all their diverse (most likely conflicting) interests, come to form a coalition to make a coordinated choice of strategies, because by doing so everybody in the coalition ends up better off than he would behaving alone (noncooperatively). Aumann and Peleg's (1960) model of non-side-payment game (NTU game) and Scarf's core nonemptiness theorem for this game (see, e.g., Scarf (1973, theorem 8.3.6, p. 211)) serve as a breakthrough in the literature. It was with this model that economists could start analyzing cooperative behavior without imposing problematic conditions on utilities, such as the cardinal nature or the transferability. Scarf's theorem is a milestone in studies of the core, a central descriptive cooperative solution.

Wilson (1978) pioneered the study of cooperative behavior in the pure exchange economy with asymmetric information. Then came a revival of research in this area around 1990, and subsequent lively contributions by many authors established the cooperative game theory with asymmetric

information as one of today's central and burgeoning research areas in economic theory and game theory. These works have been done within various frameworks, with diverse degrees of generality. The most general framework may be called a *cooperative extension of the Bayesian game*, on account of its rich structure: Like the Bayesian game it addresses players' strategy-choice in the presence of asymmetric information, but unlike the Bayesian game it allows for coordinated strategy-choice by several players. Like the non-side-payment game it explicitly formulates coalitional attainability, but unlike the non-side-payment game it allows for the influence of outsiders' behavior upon each coalition. This framework is indispensable, for example, in the analysis of an economy with organizations as production units (firms), in particular in the analysis of resource allocation mechanisms instituted in firms as superior alternatives to the incomplete market mechanism. Firms in the present-day free societies are places within which human-resource holders having private information coordinate their strategies, and these firms are interdependent in that the feasibility and implications of strategy-choice within a firm are influenced by the outsiders' strategy-choice.

Actually there have been mainly two alternative approaches to the Bayesian cooperative game: One strand is based on the view that, while the conventional noncooperative Bayesian analyses sometimes have assumed the presence of a mediator for the firm activities, there is no need for a mediator in the cooperative Bayesian analysis. Indeed, in reality, firms are operated without consulting with a mediator; the managers at various levels of the corporate hierarchy are not mediators but players in a coalition pursuing their own interests. While the principal-agent theory explains the institution of a mechanism as a solution to the mediator's optimization problem, cooperative Bayesian analysis explains it as an endogenously determined strategy bundle chosen by the insiders of the coalition. The other strand stands closer to the conventional noncooperative Bayesian analysis; it retains a mediator and gives him a major role in the coalition's execution of a strategy bundle.

The purpose of this book is to systematically present the cooperative game theory with asymmetric information – from Wilson (1978) to the current frontier, from general models to specific examples, and all the approaches that have been proposed to date. This theory is far from complete; on the contrary, there are many unsolved questions, and in fact up to now there has not been any definitive general work. Nevertheless, we decided to take on this task, hoping that clarification of the past accomplishments of the area and their appraisal will facilitate further development of the theory.

The book is organized as follows: Part I provides the key ingredients, such as the basic one-shot model and several examples (chapter 2), and the two meaningful conditions that endogenous variables (strategies) are required to satisfy: the measurability requirement (chapter 3) and the Bayesian incentive compatibility requirement (chapter 4).

Part II reviews several basic issues that have been addressed with solution concepts. The emphasis here is on several descriptive solution concepts (defined in chapter 5), although normative solution concepts that formalize efficiency are also defined (chapter 6). We report works on the comparison of two descriptive solutions, the fine core and the *ex post* core, and on the comparison of implications of the two required conditions, measurability and Bayesian incentive compatibility (chapter 7). Given the general framework, we present existence theorems for descriptive solutions (chapter 8). We also present our view on how to evaluate the existence and the nonexistence results (the end of chapter 8). We close part II with analyses of information revelation processes, that is, studies of how private information turns into public information (chapter 9).

Part III reviews studies of various versions of the core, given the specific framework of the Bayesian pure exchange economy. There are fairly general or clear-cut existence theorems, taking advantage of the specific structure of the model (chapter 10). We also look into a large economy, formulated with a nonatomic space of consumers, in the context of incomplete information (chapter 11). Actually some works on a large economy were presented in chapter 7 or will be presented in chapters 12 and 13, but the emphasis in chapter 11 is on a solution which satisfies the two major requirements (chapters 3 and 4). We provide our critical review of the recent revival of the core convergence theorem within the framework of the Bayesian pure exchange economy (chapter 12).

Part IV reviews another view on coalition formation. Specifically, we review analyses of situations in which coalitional membership is anonymous (chapter 13).