precisely, the principle plane, which is located 1.35 mm behind the corneal surface[46]) of the subject measured in meters. For example, a target 2 m away requires only 0.5 D of accommodative change, whereas a target 0.5 m away would require 2 D of accommodative change to focus on it clearly.

There are several units of measurement for vergence eye movements. Meter angle (MA), which is used in basic research, is a measure of vergence angle equal to the reciprocal of the distance of a target from the centers of rotations (about 13.5 mm behind the corneal surface[1]) of the two eyes of a binocularly viewing subject measured in meters, and thus is analogous to the diopter used for accommodation. Prism diopter (Δ), which is used in the clinic, is a unit of measure of convergence angle, where 1 Δ is equivalent to 1 cm of lateral displacement at 1 m distance, and is based on the interpupillary distance (PD). Both MA and Δ can be converted to degrees of visual angle.[64] Thus for example, a target 0.5 m in front of a subject with 6.0 cm PD has visual angles of 2 MA (= 1/0.5 m), 12 Δ (= 6 cm * 2 MA), or 6.84 deg (= 2 MA * 6 Δ/MA * 0.57 deg/Δ).

Degrees of visual angle is also used for versional eye movement measurements. The visual angle is the angle formed by the lines of sight from an eye (e.g. left eye) to the two targets (T1 and T2; Fig. 3B).

Basic Control System Concepts

A basic feedback control system block diagram is shown in Fig. 4A. The Laplace operator s is a complex variable equal to $\sigma + j\omega$, which provides information about damping and oscillatory characteristics of a system. The reason for operating in the Laplace domain is that many complicated dynamic operations in the time domain become much simpler mathematical operations

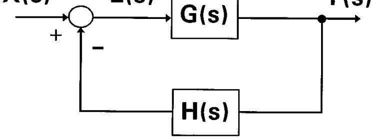

Fig. 4A Block diagram of a simple feedback control system.

in the Laplace domain. For example, when a signal is input to a block, the time domain operation consists of a convolution (i.e. a mathematical operation involving integration of time-shifted functions) between the input and the impulse response of the block. On the other hand, when the signal and the block are transformed to the Laplace domain, the transformed signal and the transfer function of the block are simply multiplied together to provide the output. To simplify this process, one can use transform pairs available in Laplace transform tables as well as a number of stability theories that have been developed. It turns out that it is easier to analyze and interpret a control system's dynamic characteristics in the Laplace than the time domain. When the analysis of the system's characteristics has been completed, the final response can be obtained simply by inverse transformation.

In Fig. 4A, $E(s)$ or the difference between the input $X(s)$ and the product of the output and feedback gain, $Y(s) * H(s)$, serve as the driving signal for the forward-loop gain $G(s)$. It can be shown that the overall transfer function is given by

$$F(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s) * H(s)}. \tag{1A}$$

For $H(s) = 1$, which is usually the case, we get

$$F(s) = \frac{G(s)}{1 + G(s)}. \tag{1B}$$

It turns out that this apparently simple equation (Eq. 1A) is the basis for much of control systems theory. It can be seen that if $G(s)H(s)$ equals -1, $F(s)$ would go to infinity and the system would become unstable. This can occur if the gain and latency values embedded in $G(s)H(s)$ are too large. Indeed, much of control systems theory involves the determination of the conditions for instability and the system modifications needed to avoid arriving at these unstable conditions.

Another useful control system property is the final value theorem, where the steady-state time domain response is given by:

$$y(t \to \infty) = \lim_{s \to 0} s\, F(s) X(s). \tag{1C}$$

The theorem states that the steady-state time domain step response can be obtained directly from the transfer function. For example, for a unit step input $X(s) = 1/s$, and transfer function $F(s) = 2/(s + 3)$, the steady-state time domain output value is

$$y(t \to \infty) = \lim_{s \to 0} s \frac{2}{s+3} \frac{1}{s} = \frac{2}{3}. \tag{1D}$$

The final value theorem forms the basis for *static* analysis of feedback control systems.

A Laplace transform pair that is often used is

$$\frac{1}{s+a} \Leftrightarrow e^{-at} u(t) \tag{1E}$$

where ⇔ designates transformation between the Laplace and time domains, $u(t)$ is the unit step function, and $a = 1/\tau$, with τ being the time constant (or the time required to reach 63% of the final step response value).

To demonstrate the significant effects of feedback, gain and latency on the dynamics of a system, six MATLAB/SIMULINK block diagrams are shown in Fig. 4B. Starting from the top: (1) a system with a latency element $e^{-T_d s}$, where the latency $T_d = 0.5$ sec, a gain element with gain = 1 and first-order transfer function of the form $1/(\tau s + 1)$, where the time constant $\tau = 0.5$ sec; (2) effect of adding a feedback connection; (3) effect of increasing gain from 1 to 1.5; (4) effect of a further increase in gain to 2.5; (5) effect of increasing latency to 1 sec; and (6) effect of further increase in latency to 2 sec. The simulation outputs (Y1−Y6) are shown in Fig. 4C starting from the upper left figure: (Y1−Y2) shows the significant change in dynamics and steady-state gain when feedback is added; (Y3−Y4) shows larger underdamped response and oscillations as gain is increased; and (Y5−Y6) shows larger underdamped response and oscillations as latency is increased.

In the accommodation and vergence eye movement systems, the latency is long and steady-state gain is high relative to the dynamic response of the system. Hence, if a simple feedback loop was used in a model of the system, the responses would consist of instability oscillations. It turns out that each of the systems solves this problem by separating its control into two parts — a fast open-loop component and a slow closed-loop component. The fast component responds to the stimulus amplitude without feedback to arrive near the desired position. The absence of any feedback (even though sensory input continues to be available) ensures stability of the initial response. Then, a slow closed-loop component takes over and reduces the small residual error to a minimum. Because the residual error is small, the gain of the slow component can be relatively low and still achieve adequate dynamic response. Yet, once these two processes are completed, the overall error would be small and the effective steady-state gain would be equivalent to a continuous feedback system with a high forward-loop gain (with its inherent instability problems). Thus, this dual-mode process achieves both rapid dynamics and small residual error without sacrificing stability.

The saccadic system also has a long latency as compared to its very fast dynamics, and thus would similarly have instability problems if it were in

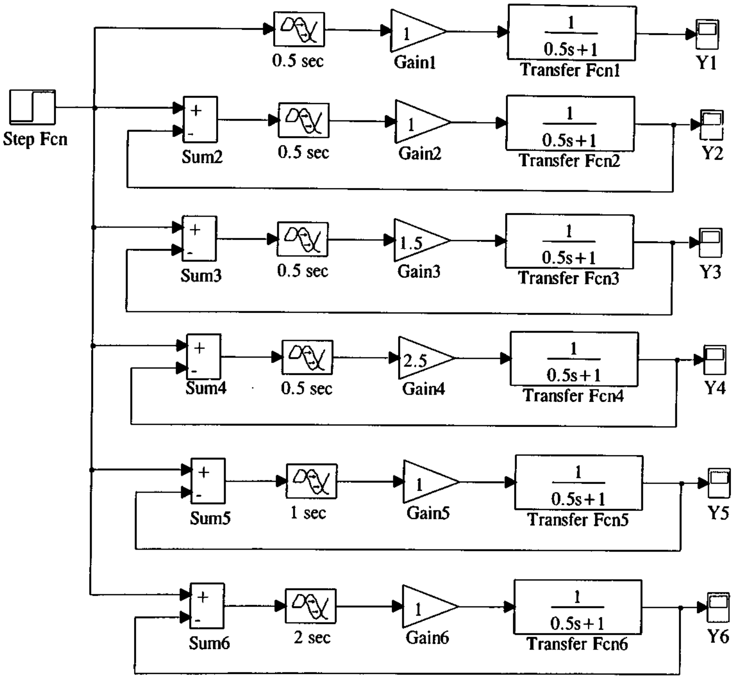

Fig. 4B Six examples of MATLAB/SIMULINK block diagrams demonstrating the effects of feedback, gain, and latency.

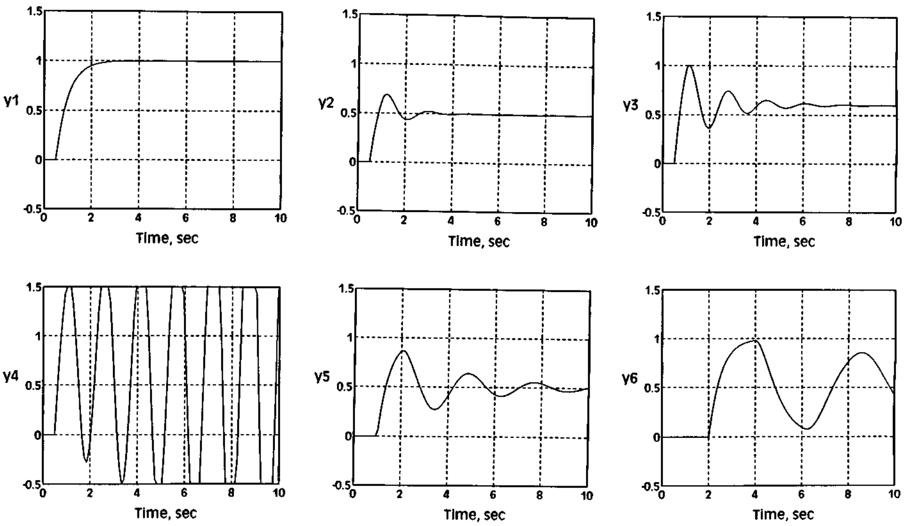

Fig. 4C Simulation results showing significant change due to feedback and increased oscillations in the response as gain and latency are increased.

a simple feedback loop. It solves this problem by responding with an initial open-loop movement, but unlike the accommodation and vergence systems, uses subsequent saccades to make corrective movements.

One of the questions often asked regarding mechanical and physiological feedback systems is “Why can’t the response be exactly equal to the stimulus?” There are two ways to answer this question. In terms of a control systems explanation, a stable feedback system will exhibit a small residual error. This can be seen upon examination of Eq. 1B, where even for a very large steady-state gain value for $G(s)$, the overall gain is still slightly less than 1. Thus, the system response cannot be exactly equal to the stimulus. In terms of a physiological explanation, it can be shown that all physical and physiological systems have a threshold for detecting a stimulus change. The region between the upper and lower thresholds is called the deadspace region. A response that falls in the middle of the deadspace region must move within the region until it finally exceeds the threshold and the signal is detected. But if the response dithers just at the boundary of a threshold, it will be able to immediately detect any stimulus change while still maintaining response accuracy. It turns out that the best solution is for the response to be at the boundary of that threshold which requires the least effort for detection, and this occurs when the response is slightly less than the stimulus. Thus, the steady-state response is seen to "lag" the stimulus, rather than being exactly equal to the stimulus. Examples of deadspace in the oculomotor system are the DOF[9] for accommodation and Panum's fusional area (PFA)[121] for vergence.

Eye Movement Measurement Techniques

Stimulus arrangement and typical experimental protocol

Both free- and instrument-space experimental protocols have been used by experimenters.[76] The free-space environment represents the normal visual scene where all the cues such as blur, disparity, overlap, perspective and shading are available to the observer. Moreover, the accommodative and vergence stimuli, based on the reciprocal of the distance of the target from the corneal plane of the observer, are equal or congruent. On the other hand, in the instrument-space environment, many of the cues are purposely removed, so that only blur and/or disparity cues are available to the observer. Moreover, the accommodative and vergence stimuli can be dissociated to provide non-congruent stimuli and a means to "dry dissect"[158] the oculomotor systems.

Accommodation measurement

Static

The static accommodative response can be measured using a Hartinger coincidence optometer. Its operation is based on the Scheiner principle.[54] Consider the situation where the subject's eye is focused at point F but also simultaneously sees the instrument alignment target A, consisting of three long vertical line segments, which is at a position indicated by the cross (Fig. 5). A prism is placed in front of the upper half, but not the lower half, of alignment target A. Thus, the rays emanating from the upper portion of target A (dotted