

Contents

<i>Preface</i>	v
1. Model of a Taut Wire	1
1.1 Deriving the PDE model	1
1.2 Balance equation	2
1.3 Boundary conditions	2
1.4 Boundary conditions (in space)	3
1.5 Initial conditions (boundary conditions in time)	4
1.6 Anything else?	5
2. The Method of Galerkin	7
2.1 Residual of the balance equation	7
2.2 Integral test of the residual	8
2.3 Test function	8
2.4 Trial function	10
2.5 Manipulation of the residuals	11
2.6 Stiffness and mass matrix	13
2.7 Piecewise linear basis functions	15
2.8 How are the Galerkin and Finite Element Methods Related	17
2.9 Numerical quadrature	18
2.10 Putting it together: system of ODE's	21
Exercises	22
3. Statics and Dynamics Examples for the Wire Model	27
3.1 Statics	28
3.2 Statics: uniform load	28

3.3	Free vibration	32
3.4	Integration of transient motion	33
3.4.1	Using built-in Matlab solver	34
3.4.2	Using the Trapezoidal integrator	35
	Exercises	38
4.	Boundary Conditions for the Model of a Taut Wire	41
4.1	Mixed essential and natural boundary conditions	42
4.2	Essential boundary conditions only	43
4.3	Natural boundary conditions only	43
4.4	Overspecified boundary conditions	44
5.	Model of Heat Conduction	49
5.1	Balance equation	49
5.2	Constitutive equation	52
5.3	Boundary conditions	53
5.3.1	On the sufficiency of boundary conditions.	54
5.4	Initial condition	55
5.5	Summary of the PDE model of heat conduction	56
	Exercises	56
6.	Galerkin Method for the Model of Heat Conduction	57
6.1	Weighted residual formulation	57
6.2	Reducing the model dimension	59
6.3	Test and trial functions: basis functions on triangulations	61
6.4	Basis functions on the standard triangle	63
6.5	Discretizing the weighted residual equation	66
6.6	Derivatives of the basis functions; Jacobian	70
6.7	Numerical integration	74
6.8	Conductivity matrix	76
6.9	Surface heat transfer matrix and load	80
	Exercises	86
7.	Steady-state Heat Conduction Solutions	89
7.1	Steady-state heat conduction equation	89
7.2	Thick-walled tube	89
7.3	Orthotropic insert	93

7.4	The T4 NAFEMS Benchmark	96
8.	Transient Heat Conduction Solutions	101
8.1	Discretization in time for transient heat conduction	101
8.2	The T3 NAFEMS Benchmark	104
8.3	Transient cooling in a shrink-fitting application	107
9.	Expanding the Library of Element Types	111
9.1	Quadratic triangle T6	112
9.2	Quadratic 1-D element L3	114
9.3	Point element P1	114
9.4	Integrating over n -dimensional domains	115
9.5	Tetrahedron T4	120
9.6	Simplex elements	122
9.7	Quadrilateral Q4	123
9.8	Hexahedron H8	124
9.9	Extracting the mesh boundary	124
	Exercises	126
10.	Discretization Error, Error Control, and Convergence	129
10.1	Interpolation errors	129
10.1.1	Interpolation error for temperature	129
10.1.2	Interpolation error for temperature gradient	132
10.1.3	Controlling the error; Convergence rate	134
10.2	Richardson extrapolation	136
10.3	The T4 NAFEMS Benchmark revisited	138
10.4	Graded meshes	139
10.5	Shrink fitting revisited	139
10.6	Representing functions by interpolation	141
	Exercises	143
11.	Model of Elastodynamics	145
11.1	Balance of linear momentum	145
11.2	Stress	147
11.2.1	Balance of angular momentum and stress symmetry.	150
11.3	Local equilibrium	152
11.3.1	Change of linear momentum	152
11.3.2	Stress divergence	152

11.3.3	All together now	156
11.4	Strains and displacements	156
11.5	Constitutive equation	159
11.6	Boundary conditions	161
11.6.1	Example: concrete dam	161
11.6.2	Example: rigid punch	162
11.6.3	Formal definition of the boundary conditions	163
11.6.4	Inadmissible “concentrated” boundary conditions	164
11.6.5	Symmetry and anti-symmetry	166
11.6.6	Example: a pure-traction problem	168
11.6.7	Example: shaft under torsion	170
11.6.8	Example: overspecified boundary conditions	172
11.7	Initial conditions	172
12.	Galerkin Formulation for Elastodynamics	175
12.1	Manipulation of the residuals	175
12.1.1	The first two steps	175
12.1.2	Step 3: Preliminaries	176
12.1.3	Step 3: The glorious conclusion	177
12.2	Method of weighted residuals as the principle of virtual work	179
12.3	Discretizing	179
12.3.1	The trial function	179
12.3.2	The test function	181
12.3.3	Producing the requisite equations	182
12.4	The discrete equations: system of ODE’s	184
12.4.1	Inertial term: Mass matrix	185
12.4.2	Body loads and traction loads	186
12.4.3	Resisting forces: Stiffness matrix	186
12.4.4	Summary of the elastodynamics ODE’s	187
12.5	Constitutive equations of linearly elastic materials	188
12.5.1	General anisotropic material	188
12.5.2	Orthotropic material	188
12.5.3	Transversely isotropic material	189
12.5.4	Isotropic material	190
12.6	Imposed (thermal) strains	191
12.7	Strain-displacement matrix	193
12.7.1	Transformation of basis	194
12.8	Stiffness matrix	197

12.9 Pure-traction problems and singular stiffness	199
Exercises	200
13. Finite Elements for True 3-D Problems	201
13.1 Modal analysis with the tetrahedron T4: the drum	201
13.2 Modal analysis with the tetrahedron T4: the composite rod	204
13.3 Tetrahedron T10	207
13.3.1 Example: the drum revisited	208
13.4 The composite rod with the tetrahedron T10	209
13.5 Static analysis with hexahedra H8 and H20	210
13.5.1 Hexahedron H8	210
13.5.2 Dilatational locking	211
13.5.3 Shear locking	214
13.5.4 Thin clamped square plate with concentrated load	215
13.5.5 Quadratic element H20	216
13.5.6 Quadratic element Q8	220
13.5.7 Pinched cylinder	221
13.5.8 Pinched sphere	222
13.5.9 Beam deflection revisited	223
13.6 Errors, validation, and verification	224
13.6.1 Verification and Prediction	226
13.6.2 Validation	227
13.6.3 Errors	227
13.6.4 Using modeling to make predictions	227
13.6.5 Using benchmarks	228
Exercises	230
14. Analyzing the Stresses	231
14.1 Singularities	231
14.2 Interpretation of stresses	234
14.3 Stress concentrations	235
15. Plane Strain, Plane Stress, and Axisymmetric Models	237
15.1 Plane strain model reduction	237
15.2 Plane stress model reduction	240
15.3 Model reduction for axial symmetry	242
15.4 Material stiffness for two-dimensional models	245

15.5 Strain-displacement matrices for two-dimensional models	246
15.6 Integration for two-dimensional models	247
15.7 Thermal strains in two-dimensional models	249
15.8 Examples	250
15.8.1 Thermal strains in a bimetallic assembly	250
15.8.2 Orthotropic balloon	254
15.9 Transient dynamic analysis	257
15.9.1 Centered difference time stepping	257
15.9.2 Example: stress wave propagation	259
Exercises	263
16. Consistency + Stability = Convergence	265
16.1 Consistency	265
16.1.1 Completeness	265
16.1.2 Compatibility	267
16.2 Stability	268
16.2.1 Conclusion	269
Exercises	270
<i>Bibliography</i>	271
<i>Index</i>	273