Chapter 1

INTRODUCTION

In recent years there has been much interest in describing the nucleus as a relativistic system. In this work we will present some of the evidence for the utility of this approach and show how a systematic application of a relativistic model of nuclear dynamics is able to resolve some long-standing puzzles in the theory of nuclear structure. We will discuss such matters as the binding energy and saturation density of nuclear matter, the effective force in nuclei, and the nucleon self-energy for bound and continuum nucleons. We also discuss the general theory of the relativistic optical model and applications to nucleon-nucleus scattering made using the relativistic impulse approximation.

The success of the relativistic analysis is based upon the use of the Dirac equation for the description of the motion of a nucleon. The justification for the use of this equation lies largely on the outstanding phenomenological success of this approach. No one has yet provided a theoretical basis for using the Dirac equation to describe the motion of large composite objects such as the nucleon. This lack of a firm theoretical base has not prevented nuclear researchers from providing an extremely useful phenomenological theory which yields a comprehensive description of many aspects of nuclear dynamics.

While it is interesting to see how the use of the Dirac equation leads to good fits to various data, it is even more interesting to understand exactly how this theory achieves its success. As we will see, one finds that the potentials which are used in the Dirac equation contain large (Lorentz) scalar and vector fields. The scalar fields enter the Dirac equation in the same way as the nucleon mass. Since these fields are quite large (~ -400 MeV), they have the effect of inducing a major reduction of the nucleon mass when the nucleon is in nuclear matter. It is the description of this change of the mass of the nucleon that is an essential element in the success of the relativistic approach. It is further necessary to understand that the vector field seen by a nucleon is large and repulsive, so that the *energy* of the nucleon in nuclear matter does not differ very much from the energy of a nucleon moving in the weak fields which appear in the standard Schroedinger description. More precisely, the dispersion relation relating the energy and momentum of a nuclear quasiparticle is similar to that of the Schroedinger theory. Therefore, the system may be said to exhibit "hidden relativity." Indeed, for decades the Schroedinger approach to nuclear structure physics provided a reasonably satisfactory model of nuclear dynamics. It is only in the last decade that the true relativistic features of the system have become apparent.

We may facilitate the understanding of the foregoing comments and the following material by considering an elementary problem. We write the Dirac equation for a nucleon in nuclear matter as

$$[\vec{\alpha}\cdot\vec{p} + \gamma^0 m_N + V(\vec{p})]\phi(\vec{p},s) = \varepsilon_{\vec{p}}\phi(\vec{p},s) \ , \tag{1.1}$$

where $V(\vec{p})$ is the potential. It will be useful to introduce the *self-energy* $\Sigma(\vec{p}) = \gamma^0 V(\vec{p})$ and rewrite this equation as

$$[\vec{\gamma}\cdot\vec{p} + m_N + \Sigma(\vec{p})]\phi(\vec{p},s) = \gamma^0\varepsilon_{\vec{p}}\phi(\vec{p},s) \ . \tag{1.2}$$

Now let us assume that the self-energy is of the form

$$\Sigma(\vec{p}) = A + \gamma^0 B \ , \tag{1.3}$$

so that we have,

$$[\vec{\gamma}\cdot\vec{p} + (m_N + A)]\phi(\vec{p},s) = [\varepsilon_{\vec{p}} - B]\gamma^0\phi(\vec{p},s) \, . \tag{1.4}$$

A positive-energy spinor soliton of this equation is

$$\phi(\vec{p},s) = \left(\frac{\tilde{m}}{\tilde{E}(\vec{p})}\right)^{\frac{1}{2}} u(\vec{p},s,\tilde{m}) \tag{1.5}$$

where,

$$u(\vec{p},s,\tilde{m}) = \left(\frac{\tilde{E}(\vec{p})+\tilde{m}}{2\tilde{m}}\right) \begin{pmatrix} \chi_s \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{\tilde{E}(\vec{p}) + \tilde{m}}\chi_s \end{pmatrix}. \tag{1.6}$$

Here $u(\vec{p},s,\tilde{m})$ is the positive-energy solution of the Dirac equation without interaction, except for the fact that the nucleon mass m_N has been replaced by $\tilde{m} = m_N + A$, and $\tilde{E}(\vec{p}) = [\vec{p}^2 + \tilde{m}^2]^{1/2}$. The normalization chosen here is

$$u^\dagger(\vec{p},s,\tilde{m})u(\vec{p},s,\tilde{m}) = \tilde{E}(\vec{p})/\tilde{m} \, , \tag{1.7}$$

so that

$$\phi^\dagger(\vec{p},s)\phi(\vec{p},s) = 1 \, . \tag{1.8}$$

We further note that the energy eigenvalue is

$$\varepsilon_{\vec{p}} = B + \sqrt{\vec{p}^2 + \tilde{m}^2} \tag{1.9}$$

$$\simeq m_N + B + A + \frac{\vec{p}^2}{2\tilde{m}} + \ldots \, . \tag{1.10}$$

Now, as we have noted, A is large and negative (~ -400 MeV) and B is large and positive (~ +300 MeV). Therefore A and B largely cancel and the dispersion relation [Eq. (1.10)] is essentially the same as that which one would find in a nonrelativistic model. We may also note the significant enhancement of the lower components in the spinor of Eq. (1.6).

This highly simplified model will be refined in the following sections. We remark that in nuclear matter the self-energy will be found to have three terms rather than the two shown in Eq. (1.3). In addition, these terms will each be momentum dependent. Further, we will see that in our construction of the self-energy operator which describes the interaction of an off-shell nucleon with a spin-zero nucleus, we will find eight terms, five of which then vanish on the passage to the nuclear matter (or no-recoil) limit (see Sec. 3.2).

Further insight into the nature of the relativistic model may be gained by calculating the expectation value of the Dirac operator $\vec{\alpha}$ for the wave function $u(\vec{p},s,\tilde{m})$. (We recall that the operator $\vec{\alpha}$ plays the role of the velocity operator in the Dirac theory.) We have

$$(\tilde{m}/\tilde{E}(\vec{p}))\ u^{\dagger}(\vec{p},s,\tilde{m})\vec{\alpha}u(\vec{p},s,\tilde{m}) = \vec{p}/\tilde{E}(\vec{p}) \tag{1.11}$$

rather than $\vec{p}/m_N$. Thus we see that there is an enhancement of the velocity (for a given momentum) of about *sixty percent*, since $m_N/\tilde{m}(0) \sim 1.56$. In the *relativistic* model, a particle at the Fermi surface in nuclear matter has a velocity of about forty percent of the velocity of light. [We here use $\tilde{m}(k_F) \simeq 628$ MeV, since $A \simeq -310$ MeV for $|\vec{p}| = k_F$. In the nonrelativistic model ($\tilde{m} = m_N$) we would have a velocity at the Fermi surface of about 0.28c.] This enhanced velocity turns out to be particularly important when calculating the strength of the spin-orbit potential felt by nucleons interacting with nuclei and, in part, accounts for the success of the relativistic impulse approximation which is discussed in Sec. 3.5.

It is important at this stage to emphasize that we are not discussing the conventional "effective mass" which has always appeared in the nonrelativistic theory of nuclei. This effective mass is defined in the usual manner,

$$\frac{1}{m^*} = \left[\frac{1}{p}\frac{d\varepsilon_{\vec{p}}}{dp}\right]_{p = k_F} \tag{1.12}$$

and corresponds to the expansion,

$$\varepsilon_{\vec{p}} = m_N + \frac{\vec{p}^{\,2}}{2m^*} + \ldots \quad . \tag{1.13}$$

It is rather striking that $\tilde{m}$ is essentially of the same size as m^* [An81c], and that leads to much confusion. In order to keep the distinction between these quantities clear we may refer to $\tilde{m}$ as the "Dirac mass" and call m^* the "effective mass". As we will see, it is the presence of $\tilde{m}$ in the theory which leads to enhanced spin-orbit forces in nuclei and also to the modification of those electromagnetic properties of the nucleon which depend explicitly upon the nucleon *velocity*. (For example, the replacement of m_N by $\tilde{m}$ will lead to an enhancement of the orbital g-factor in nuclei, since the *orbital* magnetic moment is directly related to the velocity of the nucleon. See, however, [Br85, Ku85].)

We believe that the modification of the nucleon mass in the nucleus is quite a remarkable phenomenon. In modern field theories leptons, quarks and gauge bosons obtain their mass through interactions with the vacuum, which is considered to be a complex medium, containing various kinds of condensed matter. The change of the nucleon mass in nuclear matter, or equivalently, the presence of large scalar fields, may be signalling a modification of vacuum properties inside nuclei. We will not expand upon such speculations in this work since such ideas have not been fully developed. Here we will limit ourselves to a description of the successes of the relativistic models of nuclear structure based upon the use of the Dirac equation for the description of nucleon motion.

1.1 Traditional Pictures of Nuclear Structure

Until recently our ideas concerning nuclear structure were essentially based upon an analogy with atomic physics. One considered the nucleus to be a collection of nucleons which interacted through two-body potentials. Alternatively, one could consider the potentials as arising from exchange of meson fields. (Since these mesons carried electric charge one was further led to the consideration of "meson-exchange currents." These considerations took one outside the realm of potential models; however, there appeared to be no reason to radically change the Shroedinger-based picture.) Of course, there were certain difficulties which could not be resolved in an entirely satisfactory fashion. For example, it was not really possible to calculate the binding energy and saturation density of nuclear matter starting from two-body forces obtained from the study of nucleon-nucleon scattering in free space. The program of calculating the properties of nuclei and nuclear matter from such forces was initiated by Brueckner and his collaborators about 1955. After extensive efforts by many investigators, it was clear that there was a fundamental problem in obtaining a good fit to nuclear properties in a parameter-free model. Thus in the early seventies we saw the introduction of what was called "density-dependent Hartree-Fock" theory. In the latter analysis one introduced a phenomenological density dependence into the two-body interaction in the medium. This interaction could then be adjusted to obtain a reasonable fit to nuclear charge and matter distributions, binding energies, etc. However, the introduction of density-dependent forces and various parameters took one away from the goal of calculating nuclear properties in a parameter-free theoretical scheme.

In the early seventies we also saw extensive development of the boson-exchange model of nuclear forces. In this model the nucleon-nucleon force was constructed via the exchange of mesonic fields with the quantum numbers of familiar mesons: π, ρ, ω and σ. (The scalar meson, σ, was thought to account for the attraction arising from certain two-pion exchange diagrams or virtual excitations of the delta isobar. We will not emphasize that interpretation and will have more to say about the σ field in the following discussion.) When one used the OBE potentials to calculate the properties of nuclear matter one found the standard problem: If the binding energy of the system was correct, the saturation density was too large, and if the density was correct, the system was underbound.

In the mid-seventies some new ideas were introduced which would ultimately change our view of nuclear structure. [Various individuals have made contributions which led us to believe that the Dirac equation was suitable for the description of the motion of nucleons in nuclei. We refer the reader to the historical survey presented in *Physics Today*.[1]] We, as others, were particularly impressed by the work of B.C. Clark and collaborators, who showed that nucleon-nucleus scattering could best be described by using a phenomenology based upon the use of the Dirac equation in the description of the motion of the projectile. The phenomenology that emerged was quite similar to that introduced by Walecka for the study of other problems in nuclear physics. The Walecka model involved only two mesons, σ plus ω, and used a mean-field approximation. The exchange of σ and ω mesons gave rise to large scalar and vector fields which were to be used in the Dirac equation. Such large fields were also obtained in the phenomenological studies of Clark and collaborators. [See also Mi72a, Mi72b, Mi74, Mi75.]

In an attempt to understand the phenomenological success of these models, the authors developed a parameter-free approach to these problems: Relativistic Brueckner-Hartree-Fock (RBHF) theory [An 83]. This theory was modeled after the Brueckner apporach, mentioned above. However, the Brueckner analysis was extended to include a relativistic description of nucleon motion. While not providing a complete theory, the RBHF analysis does provide a quite successful *parameter-free* model and leads to a deeper understanding of Dirac phenomenology. We comment on and contrast these approaches, Dirac phenomenology and RBHF theory, in the next section.

---

[1] *Physics Today* 37, 20 (1984).

1.2 Dirac Phenomenology and Relativistic Brueckner-Hartree-Fock Theory

The work on relativistic models may be divided into two categories. The first we will call Dirac Phenomenology [Wa 74, Ch 74, Ch 76, Ch 77, Wa 78, Br 77b, Br 78a, Br 78b, Se 78, No 78, No 79, Se 79a, Se 79b, Ja 79, Ja 80, Me 78, Ar 79, Ar 80, Ar 81a, Ar81b, Ar 82, An 81c, Cl 82, Se 85]. This category is distinguished by having several free parameters which are adjusted to fit nuclear data. The second category will be called Relativistic Brueckner-Hartree-Fock (RBHF) theory [An 80, An81a, An 81b, Ce 81a, Ce 81b, Ce 81c, Ce 82a, Ce82b, Sh 82, An 83] and is characterized as having *no free* parameters other than those introduced in fitting free-space nucleon-nucleon scattering data. Interest in the development of the RBHF approximation grew out of the successful application of Dirac phenomenology to the description of nucleon-nucleus scattering data [Cl 82]. In the application of Dirac phenomenology to the description of nucleon-nucleus scattering one usually writes the Dirac equation as

$$\left[\gamma^0 E(\vec{p}) - \vec{\gamma}\cdot\vec{p} - m_N - A(\vec{r}) - \gamma^0 B(\vec{r})\right]\psi_{\vec{p}}(\vec{r}) = 0 \tag{1.2.1}$$

where $E(\vec{p})$ is the energy of the projectile and $A(\vec{r})$ and $B(\vec{r})$ are scalar and vector potentials. These are often denoted as $U_S(r)$ and $U_0(r)$ in the literature. (In a rough approximation $\mathrm{Re}A(r)$ may be associated with exchange of scalar mesons between the nucleon and the nucleus while $\mathrm{Re}B(r)$ is associated with the exchange of vector mesons. We will see that there are important corrections to this simple description; however, it does provide a simple first orientation to the underlying concepts.) For simplicity $A(r)$ may be taken as proportional to the scalar density of the target and $B(r)$ may be taken to be proportional to the baryon density. In more refined applications a "folding model" is used which takes into account the finite range of the interaction [Ar 81b]. In the work of Walecka and collaborators concerning the properties of nuclear matter [Wa 74] and of finite nuclei [Ho 81a] it was suggested that $A \sim -400$ MeV and $B \sim 300$ MeV. These are clearly very large potentials. Since the effective central potential to be used in the Schroedinger equation is [Cl 82]

$$V(\vec{r},E) \simeq \frac{m_N}{E(\vec{p})}\left[A(\vec{r}) + \frac{E(\vec{p})}{m_N}B(\vec{r}) + \frac{A^2(\vec{r}) - B^2(\vec{r})}{2m_N} + \ldots\right] \quad (1.2.2)$$

it is clear that these large values of A and B are not inconsistent with Schroedinger potentials of a central well depth of about 50 MeV. Further, the large values of A and B are able to reproduce the magnitude of the spin-orbit potential in finite nuclei [Br 77b].

The analysis of nucleon-nucleus scattering carried forward by Clark, Mercer and Arnold [Ar 79, Ar 80, Ar 81a, Ar 81b, Me 78, Cl 82] also yields values for A and B similar to those quoted above. These authors have determined values for real and imaginary parts of A and B for $0 < T(p) < 1$ GeV and have shown that these parameters have a significant energy dependence [Cl 82].

We can now ask for the relevance of Dirac phenomenology for nuclear many-body theory. A rather direct way to approach this question is to ask for the proper description of the nucleon wave function in nuclear matter. Until the 1970's most discussions of nuclear matter used Schroedinger theory [Da 78]. In that case there is no question as to the nature of the wave function. The nucleon is described by a product of a Pauli spinor and isospinor and a plane wave,

$$\phi_{\vec{p},s,\tau}(\vec{x}) = \frac{e^{i\vec{p}\cdot\vec{x}}}{(2\pi)^{3/2}}\chi_s\chi_\tau \quad . \quad (1.2.3)$$

With the development of the one-boson exchange (OBE) model of nuclear forces [Er 74] one could perform relativistic calculations of the properties of nuclear matter. In these calculations the nucleon wave function was taken to be a spinor solution of the free Dirac equation,

$$\phi^{rel}_{\vec{p},s,\tau}(\vec{x}) = \left(\frac{m_N}{E_N(\vec{p})}\right)^{\frac{1}{2}} u(\vec{p},s)\,\frac{e^{i\vec{p}\cdot\vec{x}}}{(2\pi)^{3/2}}\chi_\tau \quad . \quad (1.2.4)$$

For example, the energy of nuclear matter could be written as

$$E_1 = \sum_s \int \frac{d\vec{p}}{(2\pi)^3}\left[\frac{m_N}{E_N(\vec{p})}\right] \bar{u}(\vec{p},s)(\vec{\gamma}\cdot\vec{p}+m_N)u(\vec{p},s)\theta(k_F-|\vec{p}|)$$

$$+\frac{1}{2}\sum_{ss'}\iint \frac{d\vec{p}}{(2\pi)^3}\frac{d\vec{p}}{(2\pi)^3}\frac{m_N}{E_N(\vec{p})}\frac{m_N}{E_N(\vec{q})}\Big(\bar{u}(\vec{p},s)\bar{u}(\vec{q},s')|$$

$$\times\ \hat{M}(1-P_{12})|u(\vec{p},s)u(\vec{q},s')\Big)\theta(k_F-|\vec{p}|)\theta(k_F-|\vec{q}|) \qquad (1.2.5)$$

where $\hat{M}$ is an appropriate reaction matrix and P_{12} is an exchange operator. In Eq. (1.2.5) and in the following we will drop any explicit reference to the isospin quantum numbers, for simplicity. As in standard Bruechner theory [Da 78], $\hat{M}$ is modified from the free nucleon-nucleon T matrix by Pauli-principle corrections as well as dispersive effects.

Now we can ask in what circumstances is Eq. (1.2.5) a reasonable approximation. We will argue that it is preferable to obtain the nucleon spinor in nuclear matter as a solution of [An 81a, Ce 81a]

$$[\vec{\gamma}\cdot\vec{p} - m_N - \Sigma(\{f(\vec{p},s)\},\vec{p})]f(\vec{p},s) = 0 \qquad (1.2.6)$$

where $\Sigma(p)$ is the nucleon self-energy. We have indicated in Eq. (1.2.6) that the self-energy is a functional of the spinor solutions, $f(\vec{p},s)$. Thus Eq. (1.2.4) is replaced by,

$$\phi^{\mathrm{rel}}_{\vec{p},s,\tau}(\vec{x}) = \left(\frac{m_N}{E_N(\vec{p})}\right)^{1/2} f(\vec{p},s)\,\frac{e^{i\vec{p}\cdot\vec{x}}}{(2\pi)^{3/2}}\chi_\tau$$

$$= \phi(\vec{p},s)\,\frac{e^{i\vec{p}\cdot\vec{x}}}{(2\pi)^{3/2}}\chi_\tau \qquad (1.2.7)$$

where we have chosen the normalization,

$$f^\dagger(\vec{p},s)f(\vec{p},s) = E_N(\vec{p})/m_N\ . \qquad (1.2.8)$$

Using various approximations we will note below, one can show that a reasonable expression for the energy of relativistic nuclear matter is given by [An 81a, An 81b],

$$E_2 = \sum_s \int \frac{d\vec{p}}{(2\pi)^3} \left[\frac{m_N}{E_N(\vec{p})} \right] \bar{f}(\vec{p},s)(\vec{\gamma} \cdot \vec{p} + m_N) f(\vec{p},s) \theta(k_F - |\vec{p}|)$$

$$+ \frac{1}{2} \sum_{ss'} \iint \frac{d\vec{p}}{(2\pi)^3} \frac{d\vec{q}}{(2\pi)^3} \left[\frac{m_N}{E_N(\vec{p})} \right] \left[\frac{m_N}{E_N(\vec{q})} \right] \Big(\bar{f}(\vec{p},s)\bar{f}(\vec{q},s') |$$

$$\times \ \hat{M}(1 - P_{12}) | f(\vec{p},s) f(\vec{q},s') \Big) \theta(k_F - |\vec{p}|)\theta(k_F - |\vec{q}|) \ . \quad (1.2.9)$$

We will specify procedures for calculation of $\hat{M}$ at a later point.

Now we can ask if the use of Eqs. (1.2.6) - (1.2.9) will yield significantly different results than the use of Eqs. (1.2.4) and (1.2.5). This matter can be related to the size of the parameter α in the expansion,

$$f(\vec{p},s) = \frac{1}{\sqrt{1 + \alpha^2(\vec{p})}} \left[u(\vec{p},s) + \alpha(\vec{p}) \sum_{s'} \langle s' | \vec{\sigma} \cdot \hat{p} | s \rangle w(\vec{p},s') \right], \quad (1.2.10)$$

where $w(\vec{p},s) = v(-\vec{p},-s)$ is a negative energy solution of the free Dirac equation. (We are here using the notation of Bjorken and Drell [Bj 64].) Equation (1.2.10) is somewhat simpler for states of definite helicity. For $\vec{p}$ along the z-axis,

$$f(\vec{p},s) = \frac{1}{\sqrt{1 + \alpha^2(\vec{p})}} \left[u(\vec{p},s) + \alpha(\vec{p})(-1)^{1/2-s} w(\vec{p},s) \right] \ . \quad (1.2.11)$$

Clearly, if α is "small" the modifications introduced in Eqs. (1.2.6)-(1.2.9) will not be important. We will see that $\alpha \sim |\vec{p}A|/(2m_N^2)$, which for $|\vec{p}| \sim k_F$, yields $\alpha_{max} \sim 0.06$ if $A \sim 400$ MeV. Thus, the very large values of A obtained from Dirac phenomenology yield values of α that could be considered to be significant. It turns out that the energy corrections are of order α^2; however, the scale is set by twice the nucleon mass. Thus we have $(2\, m_N) \langle \alpha^2 \rangle_{AVG} \sim 4$ MeV, which is a sizeable correction. Detailed calculations, to be discussed later, show that the difference of the two quantities given in Eqs. (1.2.5) and (1.2.9) is approximately,

$$\Delta E = E_2 - E_1 \simeq 3.6(\rho/\rho_0)^{2.4}\text{MeV} . \tag{1.2.12}$$

[More precise estimates will be given in Sec. 2.4.] Here ρ_0 is the density of nuclear matter, $\rho_0 = 0.17\ \text{fm}^{-3}$. The modification of the energy exhibited in Eq. (1.2.12) leads to *major modifications in the predicted saturation density and binding energy of nuclear matter* [An 80, An 81b].

We see that α^2 is proportional to A^2 in first approximation. Therefore, the very large values of A found from studies in Dirac phenomenology suggest that one needs to reconsider the calculations of various nuclear properties. In Chap. 2 we will discuss the binding energy and saturation density of nuclear matter as well as the effective interaction in nuclei. In Chap. 3 we will consider the nucleon self-energy operator for continuum nucleons. At that point we can make contact with the extensive phenomenological studies of the nucleon optical potential which make use of the Dirac equation [Ar 79, Ar 80, Ar 81a, Ar 81b, Me 78, Cl 82].

We will see that the relativistic theory (RBHF) provides a reasonable description of the saturation density of nuclear matter. It is gratifying to note that the same mechanism which is responsible for saturation in the RBHF model also explains the extraordinary density dependence of the *effective* nucleon-nucleon interaction [Ri 73, Ri74, Sp 77, Ri 78]. This density dependence appears in the part of the nucleon-nucleon interaction that is independent of spin and isospin. (If one uses the Migdal parameterization [Mi 67, Mi 68] of the effective interaction this density dependence appears in the parameter F_0 — see Sec. 2.5.)

Finally we note that the RBHF model provides a good description of the parameters of the relativistic optical model. The calculated imaginary parts of $A(r)$ and $B(r)$ agree well with those determined in phenomenological studies [Cl 82]. The calculated values of ReA(r) and ReB(r) appear about 25 percent too small when compared with the corresponding phenomenological parameters [Cl 82]. However, there are additional terms in the self-energy other than those indicated in Eq. (1.2.1) and a careful analysis is needed before one can compare the theoretical and phenomenological values of the optical potential parameters. It appears that the phenomenological potentials are *effective* potentials whose strengths are adjusted to compensate for the simplified form of the phenomenological potential [Ce 85 a]. Therefore, in the following discussion, we will usually denote the phenomenological potentials by $U_s(r)$ and $U_0(r)$ and reserve the notation A and B for potentials determined using RBHF theory.

1.3 Applications of Dirac Phenomenology

We will not attempt to review the very extensive literature in the field of Dirac phenomenology. However, we will comment on some recent work.

A rather detailed study of the properties of finite nuclei has been published [Ho81a]. As characteristic of research in this area, the work described in [Ho81a] contains several free parameters. The resulting fits to the properties of finite nuclei are as good as those obtained in the density-dependent Hartree-Fock analysis. Some application of the scalar and vector potentials determined in [Ho81a] have been made by Clark and collaborators [Cl82]. In these applications to the study of p-^{40}C scattering over a wide range of energies, imaginary scalar and vector potentials were added to the real potentials obtained in [Ho81a]. (The imaginary potentials were taken to be two-parameter fermi shapes for energies above 50 MeV.) In addition, the real scalar and vector potentials were shown to have a strong energy dependence and thus only the radial shape of the potentials of [Ho81a] was retained. The real scalar and vector potentials of [Ho81a] were each multiplied by a phenomenological energy-dependent constant. In the end there are eight phenomenological constants to be determined in the analysis of p-^{40}Ca scattering.

Studies of p-^{40}Ca were made at 26.3 MeV, 181 MeV, 497.5 MeV and 800 MeV [C182]. Satisfactory fits to the data were obtained at these energies. The agreement with cross sections and analyzing power data at 181, 500 and 800 MeV was very good. The spin-rotation function, $Q(\theta)$, was predicted on the basis of the fit to the differential cross section and analyzing-power data at 500 MeV, and was found to be in good agreement with the experimental data. In Figs. 1.1 - 1.3 we show the results of the analysis at 497.5 MeV. Standard 12 -parameter optical model treatments which make use of the Schroedinger equation are unable to provide a fit to the data for $Q(\theta)$ [Cl 82].

Studies using Dirac phenomenology require the introduction of several free parameters. However, the optical model studies have shown, in a rather convincing fashion, that it is preferable to parameterize the

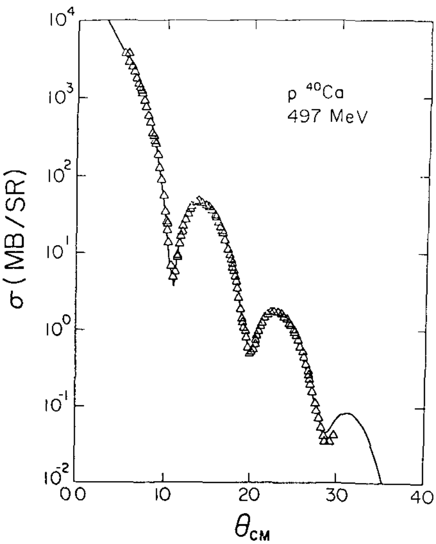

Fig. 1.1 Calculated elastic p-^{40}Ca cross sections at 497.5 MeV. (This figure appears as Fig. 9 in [C182].) The calculation is made using a Dirac equation containing phenomenological (complex) scalar and vector potentials.

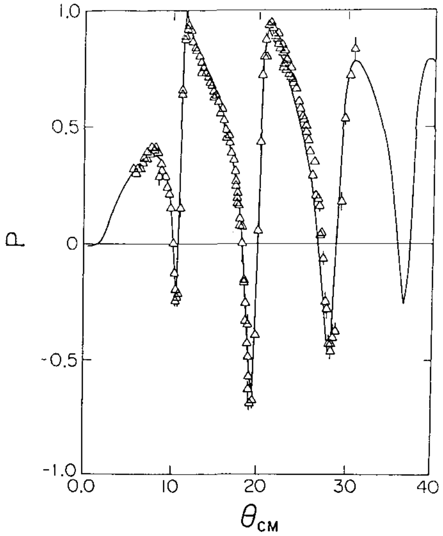

Fig. 1.2. Calculated elastic p-^{40}Ca analyzing power at 497.5 MeV. (This figure appears as Fig. 10 in [Cl82].) See caption to Fig. 1.1.

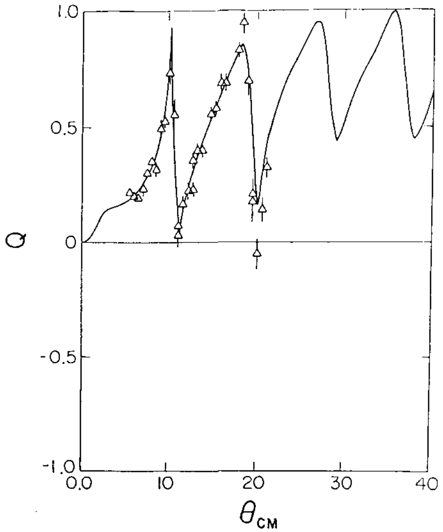

Fig. 1.3. Calculated values of the spin-rotation function for elastic p-^{40}Ca scattering. (This figure appears as Fig. 11 in [Cl82] and represents a prediction of the values of Q(θ) based upon the parameters determined in the fit to the differential cross section and analyzing power data of [Ho81d]. This data for Q(θ) are from [Ra81].)

optical potential at the level of the Dirac equation. The potentials inserted in the Dirac equation are of a rather simple structure. The corresponding Schroedinger potentials may be identified by reducing the Dirac equation to an equivalent Schroedinger equation. The potentials which appear in this Schroedinger equation are very complicated and contain terms both linear and quadratic in the target density.

Further information concerning Dirac phenomenology may be found in [Ja 83, Bo 81a, Bo 81b, Bo 81c, Bo 81d, Bo 82, Ei 81, Ho 81c, Lo 81] and in the references already listed in Sec. 1.2. However, a major triumph followed the developments described above. Very good fits to scattering data at higher energies ($E \gtrsim 400$ MeV) were obtained in a *parameter-free* model which made use of the relativistic impulse approximation mentioned earlier. We now turn to a discussion of that approximation.

1.4 The Relativistic Impulse Approximation

Stimulated by the success of Dirac phenomenology, Wallace, Shepard, McNeil and others [Cl 83, Sh 83, Mc 83a] developed a relativistic form of the impulse approximation. The impulse approximation for the construction of the nuclear optical potential had been very extensively developed within the context of Schroedinger dynamics. The leading term of the optical potential could be obtained in the so-called "$t\rho$" approximation [Ke 59]. This name followed from the fact that the optical potential could be approximated as

$$V_{opt}(\vec{q},E) \simeq t(\vec{q},E)\rho(\vec{q}) \ , \quad \text{with } \rho(0) = A \ .$$

Here $t(\vec{q},E)$ is the nucleon-nucleon scattering amplitude, $\vec{q}$ is the momentum transfer, and $\rho(\vec{q})$ is a nuclear form factor. The approximation in question follows from the evaluation of a diagram such as Fig. 1.4. Here t denotes the nucleon-nucleon scattering amplitude, and the lower part of the diagram can be related to the nuclear density matrix. If one assumes that at relatively high energies ($E > 400$ MeV), t depends only upon the energy and momentum transfer one obtains the simple result given above. [See Appendix A.3.]

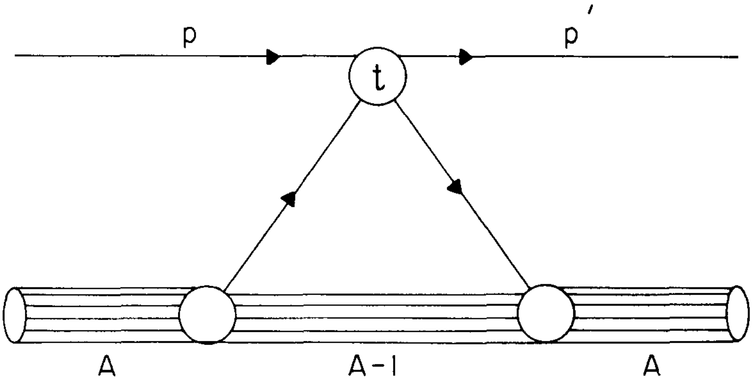

Fig. 1.4.

The generalization to the relativistic case is immediate and is discussed in more detailed in Sec. 3.5. Essentially one needs the nucleon-nucleon scattering amplitude for *off-mass-shell* nucleons and a model for the density matrix of the target, considered as a relativistic system. Relativistic Hartree models of various nuclei have been developed and these models yield the relevant density matrices. An off-mass-shell model of the nucleon-nucleon scattering amplitude may be obtained from a specific off-shell extrapolation of on-shell amplitudes, or alternatively, a specific dynamical model may be used. As we will see (in Sec. 3.6) quite remarkable fits to nucleon-nucleus scattering data may be achieved using the relativistic impulse approximation. These parameter-free calculations represent a natural extension of the methods used at lower energies in the development of the RBHF theory. [An introduction to the use of field-theoretic methods in relativistic nuclear structure physics is presented in the Appendices.]

1.5 Importance of Scalar Fields in Nuclear and Particle Physics

We have stressed in this Introduction that the presence of large scalar fields in the nucleus leads to the need for a relativistic description. Scalar fields, which have the quantum numbers of the vacuum, also play an important role in elementary particle physics, where they are often called *Higgs fields*. These Higgs fields are usually assumed to have non-zero vacuum expectation values and therefore provide a mechanism for giving mass to gauge bosons, quarks and leptons. For example, the electron, which is the most familiar lepton, receives its mass through interaction with a Higgs field in the theory of electro-weak interactions. This mass is further renormalized through the interaction of the electrons with the electromagnetic field. Now we can note that the electron field is expanded in normal modes for the purpose of quantization, and in this expansion one uses spinor wave functions for the electron parameterized by the experimentally determined value of the mass.

Now when we consider, for convenience, a field theory of interacting (point-like) nucleons and mesons, one can pose the question of what mass to use when expanding the field operator in normal modes. The relativistic analysis given here suggests that the correct mass parameter is found from the solution of the equation,

$$\tilde{m}(\vec{p}) = m_N + \frac{1}{4}\,\mathrm{Tr}\Sigma(\vec{p},\tilde{m}(\vec{p}),\ k_F) \tag{1.5.1}$$

in nuclear matter. Here we have indicated the dependence of the self-energy operator on $\tilde{m}$ and the density of the medium. Further, we can note that these studies of relativistic nuclear physics represent the first indication that we are able to observe the modification of a fundamental quantity such as the mass of an "elementary particle." Clearly a deeper understanding of this phenomenon should give us a better understanding of QCD, the theory of strong interactions.

We may summarize some of the content of the Introduction in Fig. 1.5. In Fig. 1.5a we indicate the density profile of a large nucleus such as lead. In Fig. 1.5b we show, in a schematic fashion, the large scalar and vector fields, $A(r)$ and $B(r)$, that would influence the motion of a nucleon. In Fig. 1.5c we show that the *energy* of a nucleon is modified only to a small extent; however, in Fig. 1.5d we show that the nucleon mass parameter undergoes a major modification. As we have stressed in this discussion, it is important to keep the contrast between Fig. 1.5c and Fig. 1.5d in mind.

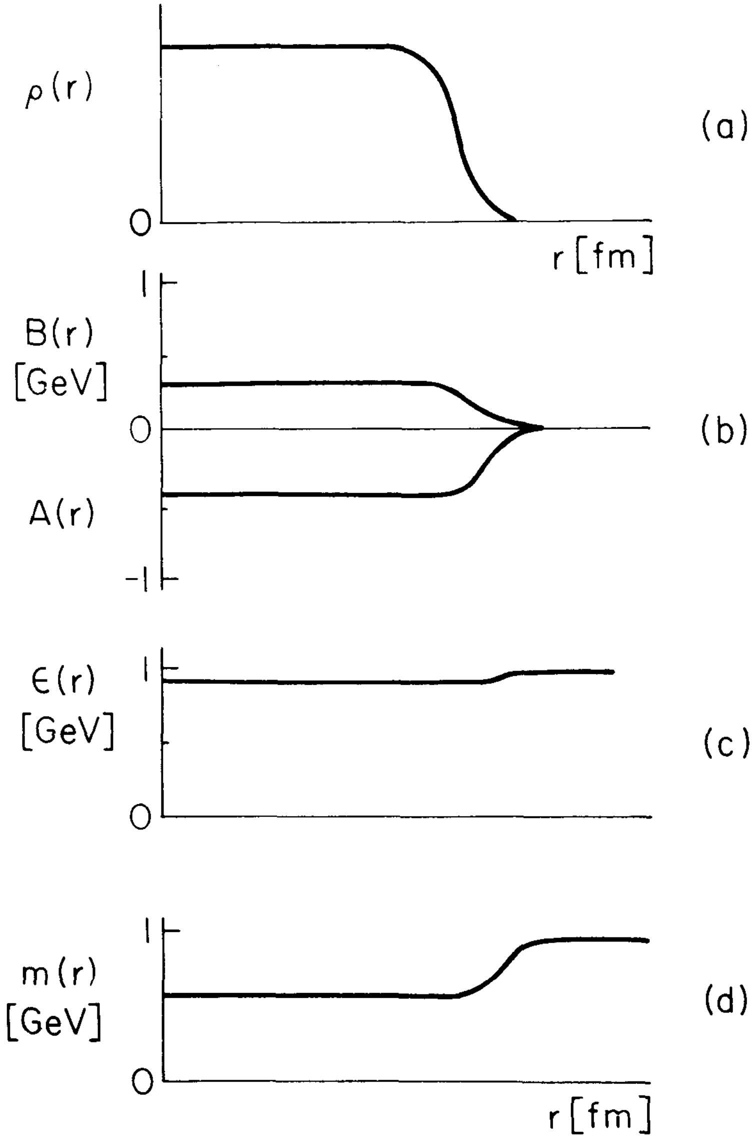

Fig. 1.5.