# FOREWORD

Why a book on Landau's theory of phase transitions ? To many physicists working in the field of phase transitions, this question will appear as doubly relevant. Indeed, why describe in detail the foundations and consequences of a theory whose basic hypothesis (the absence of singularity in the transition free-energy), and whose essential physical result (the specification of a critical behaviour) have been known for 40 years to be questionable ? Why, on the other hand, restrict to this phenomenological approach, at a time when microscopic models can be handled by various theoretical and numerical methods, and provide a "royal way" to the investigation of phase transitions ?

The existence of a satisfactory answer to these questions is attested by the fact that, ever since its formulation, Landau's theory has been used without interruption as the theoretical background of many studies of systems undergoing phase transitions. More strikingly, it is in the last 15 years, after the advent of the modern statistical theory of critical phenomena, that the utility of Landau's theory has been demonstrated most clearly, when it has been applied to the intricate patterns of transitions observed in the structural, magnetic, and liquid-crystalline systems, and more recently, to the investigation of the stabilities and properties of the incommensurate phases, and of the icosahedral quasi-crystalline phases.

There are several reasons to this persistent use of Landau's theory. A first set of reasons is related to the fact that the objected lack of validity of the basic assumptions and results of Landau's theory is not of a clearcut nature. Thus, the symmetry aspects which constitute an important part of the theory, are rigorous. Besides, there are classes of systems, governed by long range interactions (e.g. elastic interactions) for which the critical behaviour is expected to be correctly described by Landau's theory. More significantly, the temperature range adjacent to the transitions, in which the behaviour of a system is dominated by the fluctuations (and in which, accordingly Landau's theory fails), usually constitutes a small fraction of the temperature range of experimental interest. In most of the latter range, the Landau theory is an adequate tool to investigate the physical behaviour of the system.

Another important set of reasons pertains to the possibility of manipulating, through a mathematically simple and flexible theory, the complicated degrees of freedom which describe the states of real systems (e.g. sets of collective atomic displacements, or intricate spin configurations). Likewise, one has the possibility of relating simply to eachother a variety of physical properties (mechanical, optical, lattice-dynamical, structural,...) whose microscopic description would be very difficult.

The mathematical simplicity of the theory is a consequence of the clever manner by which Landau defines the order-parameter through the substitution of a small set of scalar (spatially uniform) quantities, to a set of functions having rapid variations at the atomic level. This substitution which appears as a "trick" of mere mathematical convenience, has the important consequence of permitting the description of the evolution of a complex spatial configuration of particles by means of an ordinary polynomial expansion. This trick also gives to the order-parameter a duality of meanings. As a spatially uniform quantity (or a smoothly varying one, in certain cases) it can be considered as of macroscopic nature. On the other hand, the functions it substitutes are clearly of microscopic nature. At choice, one can put the accent on one interpretation or the other (e.g. on the dielectric polarization of a ferroelectric crystal, or on the structural changes and lattice dynamical mode related to the polarization).

The flexibility of the theory resides in its modular character. Aside from the primary order-parameter, additional degrees of freedom can be incorporated in the theory as measure as the acquisition of the experimental data requires interpreting a larger set of results. For instance, in the study of a crystalline transition, once the primary order-parameter is given a sense in terms of atomic displacements, one can focus successively on the anomalies induced by the considered transition in the thermal expansion, the optical properties, the vibrational atomic spectrum, etc... In this view one will add terms in the Landau free-energy respectively corresponding to the mechanical deformations, to the dielectric polarization, to other collective atomic displacements, etc...

The only rigid feature of the theory is its symmetry framework which imposes the form of the interactions between the various degrees of freedom, and the number of adjustable phenomenological coefficients. The form of the interactions determines the relationship between the laws governing the behaviour of the various physical quantities. The explanatory power of Landau's theory resides in the checking of the overall consistency of the observed laws.

Finally, an additional reason of consideration of Landau's theory is its specific status in respect to the statistical theory of critical phenomena (Wilson's theory). From this standpoint, Landau's theory appears as a necessary point of passage, and also as a tool. On the one hand, it is the order-parameter defined by Landau's theory whose fluctuations give rise to a singularity at the transition point. Accordingly, it is on this set of degrees of freedom that the statistical theory operates. On the other hand, Landau's theory provides the rules for constructing the effective Hamiltonian density, function of the order-parameter and of the secondary degrees of freedom, on which the renormalization-group transformations act.

The contents of this book stems from three different objectives. In the first place, it is an introduction to the basic principles and techniques of Landau's theory, which is intended for teaching purposes. In this spirit, it includes an introduction to the peripheric group-theoretical and crystallographic concepts required to work out the theory. This part of the book is an expanded version of courses taught by the authors in various circumstances. Chapter I is a self-contained, simplified, introduction to the basic aspects of Landau's theory, which is well adapted to a teaching at the undergraduate level. The first paragraphs of chapter II constitute a complete presentation of the theory. They involve a thorough discussion of the starting assumptions and an explicit decomposition of the steps of the argumentation. The same pedagogical purpose has presided over the writing of chapter III, of the two first paragraphs of chapter IV, and of the four first paragraphs of chapters VI and VII. These chapters are respectively devoted to the applications of Landau's theory to structural transitions, to first-order transitions, and to magnetic and liquid-crystalline systems. Their contents is rooted in courses given at the graduate level.

A second purpose of the book is to provide the practical "recipes" for applying Landau's theory to complex systems. In this view, each element of the method is illustrated by several examples and the intermediate steps of many calculations are explicitely reproduced. Thus, one can find the constructions of the matrices of the order-parameter representation, or corepresentation (chaps. II, III, V and VI), the construction of the Landau free-energy (chaps. II and V), the description of the procedures of its minimization, and the method of identification of the low symmetry group for structural (chap. III) magnetic (chap. VI) and liquid crystal (chap. VI) systems. The procedure of application of the Landau criterion (chap. II) and of the Lifschitz one (chap. III) are exposed in details. For incommensurate systems (chap. V), an extensive description is given of the construction of the Lifschitz-invariant and other spatially dispersive terms, as well as the resolution of the equations relative to the standard situation of a two-component order-parameter. In the chapter devoted to liquid crystals, we have adopted a unified description of the various types of transitions occuring in these systems while existing theories often derive from a variety of approaches (chap. VII).

The last objective of the book is to incorporate the developments which have arisen in the last 15 years from the extensive application of the theory to a variety of physical systems. These developments involve several aspects. On the one hand, certain bases of the theory itself have been discussed. The meaning of the Lifschitz criterion has been analyzed by a number of authors and substantially clarified through the study of incommensurate systems. Its initially derivation by Lifschitz (chap. V) has been replaced by other derivations, physically more transparent, and mathematically more correct, though more complex (chap. III). Conversely, it has been understood that Lifschitz's derivation provided a method for the study of incommensurate systems (chap. V).

The second aspect of progress concerns the specification of the essential symmetries underlying the Landau theory, through the replacement of groups acting in the physical space, by groups acting in the order parameter space. One has been able, by this means, to express in a more efficient way the intrinsic symmetry of the order-parameter, the symmetry of the truncated free-energy expansion, and the characteristics of the symmetry breaking across the transition. New procedures of minimization of the free-energy have been based on the consideration of these essential symmetries. These methods have clarified the nature of the mathematical problem set by the Landau theory : find the absolute minimum of a $m^{th}$ degree polynomial in several variables, i) having a local maximum at the origin, ii) positive and infinite at infinity in any direction, iii) whose extrema have a symmetry-specified degeneracy, and iiii) which possesses obligatory extrema along symmetry directions in the order-parameter space (chap. II, paragraph 4).

Finally, a large part of the book is devoted to the systems, already mentioned above, which have been studied, in recent years, by means of Landau's theory : continuous or discontinuous structural transitions involving coupling between several relevant degrees of freedom (chaps III and IV) magnetic transitions (chap. VI), transitions in liquid-crystals (chap. VII), incommensurate phases (chap. V). We have also outline the principles of the application of the theory to icosahedral phases and to defects (chap. VIII).

In certain chapters, we had the choice between various distinct approaches. It is worth pointing out that the theory of first-order transitions is essentially inspired by the works of Gufan and co-workers (chap. IV). The theories of magnetic and liquid crystal systems have respectively their roots in methods elaborated by Dzyaloshinski and by Indenbom and coworkers (chaps VI and VII).

The multiplicity of objectives pursued has the consequence that the different chapters or paragraphs are not treated evenly. In certain parts of the book, each statement is justified, while in others, dealing with more recently developped fields, the reader is directed, for complete justification to appropriate reference works. The latter situation will be found, in particular, in large fractions of chapters IV-VIII.

In writing this book, we feel indebted to a number of colleagues. We are especially greatful to Louis Michel. From discussions with him we have learned most of the considerations pertaining to the essential symmetries of the Landau theory, which are included in the book. We have also benefitted from meeting several times Yu M. Gufan and V.P. Dimitriev who have shared with us their deep understanding of the physical implications of the Landau theory. The enlightening explanations of E. Brézin have been very helpful to clarify our view of the situation of Landau's theory from the standpoint of statistical physics. We had stimulating discussions with several experts in the handling of Landau's theory, namely N. Boccara, V. Dvorak, and A.P. Levanyuk.

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