Preface

During the last few years, considerable interest has been focused on a complex of physical ideas that share a common mathematical theme, the concept of holonomy. The recent flurry of activity began in 1984 with a paper by Michael Berry. He showed that the adiabatic evolution of energy eigenfunctions, with respect to a time-dependent quantum Hamiltonian $H(t)$, contains a phase of deeply geometrical origin (now known as "Berry's phase") in addition to the familiar dynamical phase

$$\exp -\frac{i}{\hbar}\int E(t)\,dt\,.$$

The additional phase approaches a finite, non-zero limit as the the Hamiltonian is taken more and more slowly around a closed path in its parameter space. Berry's observation, although basically elementary, seems to be quite profound. Multiplicative phases—or more generally group transformations—with similar mathematical origins have been identified and found to be important in a startling variety of physical contexts, ranging from nuclear magnetic resonace to low Reynolds number hydrodynamics to quantum field theory. It now seems clear that Berry captured a particularly fruitful concept, of wide applicability.

There are several reasons for the impact of Berry's work. Of course, the inherent universality and beauty of geometric phases has played a role, but it is also worth mentioning some of the extrinsic factors which made it "the right concept at the right time." One factor was undoubtedly surprise—the surprise of the physics community that such a simple and fundamental aspect of the adiabatic theorem had been overlooked for so many years. Another reason for its impact was the emergence of gauge theories of the interactions of elementary particles. Many gauge theoretic ideas appear in the study of geometric phases, unencumbered by the complexities usually associated with relativistic quantum field theory. Conversely, ideas associated with geometric phases clarify some subtle issues in quantum field theory—as we shall find in Chapters 5 and 7. In an era of increasing separation between everyday reality and the more theoretical branches of physics, it has been refreshing and comforting to come across a concept that both helps to explain some of the more abstruse ideas of quantum field theory, and leads to effects that can be readily measured in a laboratory.

Any judgement as to the value of an essentially mathematical concept, proposed for use as a tool in physics, should be at least partly based on its usefulness in practice. It is all too easy to believe that merely by adopting a new language one begins to make novel observations, but we trust that a perusal of the the contents of this book will suffice to show that many genuinely new insights have been gained. Although it is at present used on a relatively modest scale, we believe that the concept of a geometric phase, repeating the history of the group concept, will eventually find so many realizations and applications in physics, that it will repay study for its own sake, and become part of the *lingua franca.*

The immediate origin of this book was a workshop held on "Non-integrable Phases in Dynamical Systems" in Minneapolis on 1-3 October 1987, under the aegis of the new Theoretical Physics Institute. The enthusiastic response of the participants, and the variety and quality of work presented, led us to think that a book on the subject would make a useful addition to the physics literature. The present volume contains reprints of papers we think are particularly important or instructive, together with several contributions which have not appeared in print before. The articles are arranged by subject in nine chapters, each of which begins with an introduction where we attempt to weave the varied material into a reasonably coherent whole.

We do not purport to have made a comprehensive collection of relevant articles. Our choice of papers is more a function of our own particular interests and ignorance than anything else. Nevertheless, we hope that our book will serve as a useful introduction to an emerging field, and will stimulate its readers to seek out further material in their own areas of interest.

We are grateful to Michael Berry for his superb introductory survey and Daniel Arovas for his comprehensive article on fractional statistics and the fractional quantum Hall effect. We also wish to thank our editor, P.H. Tham, without whose hard work and assistance this book would not have been completed. The cover photo, "Calcutta Staircase 1988," was taken by Catherine Shapere.

A Reader's Guide

Probably only the most adventurous readers will be motivated to read every chapter of this book, but many readers may be interested in browsing through unfamiliar territory. As an aid to field theorists who want to understand how non–Abelian gauge potentials apply to molecular systems, and to assist physical chemists in appreciating the connection between the molecular Aharonov–Bohm effect and gauge anomalies, we have prepared introductions to each of the chapters. The introductions serve several purposes. They provide some elementary background to topics covered in the chapter, touching at least briefly on each included article. They also try to put each chapter in perspective relative to the rest of the book, and to suggest further directions for research, when possible. It is our hope that by bringing together applications of geometric phases from a variety of fields, this book will inspire continued cross–fertilization between widely separated areas of physics.

Very roughly, the chapter dependence is as follows:

$$\begin{array}{ccccc} & & 1 & & \\ & & \downarrow & & \\ & & 2 & & \\ & & \downarrow & & \\ 4 & \leftarrow & 3 & \rightarrow & 8 \\ & & \downarrow & \searrow & \\ 6 & \leftarrow & 5 & & 9 \\ & & \downarrow & & \\ & & 7 & & \end{array}$$

The first four chapters are of general interest. The first chapter includes two survey articles, by Berry [1.1] * and Jackiw [1.2], which we recommend to all readers. Both articles are relatively non–technical, and should help orient readers to the following three chapters. In addition, Berry's article includes original material on the natural metric on the projective Hilbert bundle and a detailed study of non-adiabatic corrections to the phase precession of the classical pendulum.

Chapter 2 contains some of the pre–1984 material which anticipated Berry's work on the quantal adiabatic phase, drawn from molecular physics and optics. This historical material is important and interesting in its own right—indeed, Pancharatnam's thirty–year–old paper on phase shifts of polarized light [2.1] has laid the groundwork for some modern optics experiments to measure geometric phases [4.2], and provides the basis for a recent extension of Berry's phase to non–closed paths [3.5]. The development over twenty years of phase concepts in molecular physics led to the use of gauge potentials in Born–Oppenheimer Hamiltonians several years before Berry's paper, and has borne a rich field of continuing activity.

* The notation $[M.N]$ refers to the Nth article in chapter M.

The general foundations of our subject are laid out in Chapter 3. It contains Berry's original paper [3.1] and covers many subsequent extensions and elaborations of Berry's phase, such as Wilczek and Zee's non-abelian phase for degenerate Hamiltonians [3.3] and Aharonov and Anandan's phase for general cyclic evolution [3.4]. Its introduction includes, in an appendix, a pedagogical discussion of the mathematical context of Berry's phase, which may provide useful background for some of the more mathematical articles. The final article [3.7], which has not appeared previously, is a general discussion of the Born–Oppenheimer approximation and its field theory analogues, with phase effects and non-adiabatic corrections taken into account.

Basic applications of Berry's phase are treated in the following chapter, with articles drawn from optics, magnetic resonance, and molecular and atomic physics, from both the experimental and the theoretical literature. NMR and optics have provided some of the most successful tests of Berry's phase in macroscopic systems to date. In the fully quantum mechanical context of molecular physics, phase effects can lead to energy splittings and can shift quantum numbers, that have been observed experimentally.

The remaining five chapters are concerned with more specialized applications. Chapters 5, 6, and 7, respectively on fractional statistics, the quantized Hall effect, and anomalies and Wess–Zumino terms, are about geometric phases in many-body systems and quantum field theories. All three contain extensive introductions to aid the uninitiated. We would recommend reading Chapter 5 first, since the concept of fractional statistics plays a fundamental role in the theory of the fractional Hall effect, and is closely associated with Wess–Zumino terms. In fact, the boundaries between these three chapters are not too sharply defined—for instance, the review article by Arovas contains much general material on fractional statistics, although its main focus is on the fractional Hall effect. Also, these chapters touch on several topics which are not evident from their titles. The chapter on fractional statistics includes a paper by Laughlin on high-temperature superconductivity, and Chapter 6, on the quantized Hall effect, contains articles on two other two-dimensional systems—a network of current loops enclosing magnetic flux, and a Bloch electron in a in a transverse magnetic field.

Geometric phases also appear in classical systems. Hannay's angles are classical correspondants of Berry's phase, and can appear in any classical system described by action–angle variables, in response to adiabatic variation of the Hamiltonian. Another type of classical phase occurs in describing the motion of deformable bodies, which is especially useful in studying systems that are invariant under reparameterizations of time. In particular, the motion of a self–propelled body at low Reynolds number and the rotation of a self–deforming body in space can be described in terms of a gauge field over the space of shapes. These examples are all discussed in Chapter 8.

Finally, the last chapter contains Berry's elegant paper on higher–order corrections to the adiabatic approximation. It is our belief that there is much room for further research in this area. Indeed, the subject of geometric phases in physics is far from closed, so perhaps it is fitting that we end on an unresolved note.