Chapter one

INTRODUCTION

1.1 Why Quantum Chromodynamics?

It is an article of faith that quantum chromodynamics (QCD) is the fundamental theory of the strong interactions. At high energies and large momentum transfer, where asymptotic freedom validates perturbation theory, there are impressive successes. From such experiments can be deduced the generations of quarks, their masses, charges and quantum numbers.

It is in the realm of low energy (GeV), large scale (fm) physics that the predictive power of QCD falters. The nonlinearity of the theory has presented theorists with a formidable task which has not yet yielded to analytical treatment, although vital qualitative features have emerged. On the other hand, lattice gauge calculations provide a nonperturbative numerical approach which has long held the promise of providing reliable, detailed solutions. Unfortunately, lattice calculations have proved to be extremely expensive in computation time and the results, to date, have been of limited accuracy and utility. Even the strength of the linear component of the confinement potential is not cleanly predicted. since it has not yet been possible to carry the calculations into the region of linearity; the linear component can only be extracted by assuming an explicit form for the potential and then fitting the calculations to that form. The parameters of the assumed confinement potential can be extracted empirically from heavy quark spectroscopy, and the identification of the strength of the linear term is often used to set the scale of lattice calculations. There is every reason to expect further improvements in lattice calculations as computer facilities expand, as the inclusion of dynamical quarks becomes more reliable, and more extensive analysis of the "data" becomes available.

QCD is part of the standard model, which incorporates the leptons and fields of electroweak interactions. There have been as yet no experimental violations of the standard model, but there do remain theoretical problems, and it is certainly incomplete. Among other things, it gives no mechanism for determining the various quark and lepton masses, the mixing phase angles, nor the couplings to the Bose fields. Solutions to these problems are among the goals of superstring theory. Here we will restrict ourselves to the modeling of QCD.

Why QCD? Because *it is the only game in town.*

1.2 The QCD Lagrangian

QCD is the marriage of two concepts in particle physics: the quark (parton) model, and the theory of local gauge fields. Quarks were introduced by Gell-Mann (1964) and Zweig (1964) as a purely mathematical construct to provide a framework for the SU(3) classification of elementary particles which in turn was an outgrowth of the eight-fold way. In spite of early resistance, experiments bestowed on the objects a reality of their own, and this in turn demanded the addition of a new quantum number, color, to conform to the Pauli exclusion principle. The development of Yang-Mills (1954) theory, based on the requirement of local gauge invariance, led to the introduction of gauge fields—gluons—which became the force fields, interacting with Fermions and among themselves.

The basic QCD Lagrangian density is

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu} D_{\mu} - m)\psi - \tfrac{1}{4} F^{c}_{\mu\nu} F^{\mu\nu}_{c} \tag{1.1}$$

plus Higgs bosons and appropriate counter terms. The summation convention for repeated upper-lower indices of any kind is assumed. The various constituents have the following meaning:

The quark mass m is a diagonal matrix. Since free quarks are not observed, one can only speak of quark masses in regions of confinement or asymptotic freedom. *Current* quark masses are used in relativistic bag and soliton calculations; *constituent* quark masses are used in nonrelativistic potential models. Current masses for the light, u and d, quarks are very small, while the corresponding constituent masses are roughly one-third of a nucleon mass. The larger constituent mass can be ascribed to the energy of confinement. For example, a massless quark in an MIT bag of radius R is rather like a nonrelativistic quark of mass $2.043/R$, which is the energy of the lowest bag eigenvalue. Clearly any quoted quark masses are model dependent. Table 1.1 gives approximate quark masses along with other properties.

Throughout this book, we follow the notation of Bjorken and Drell (1964). We use units where $\hbar = c = 1$. Energy then has units of inverse length and the units are sometimes quoted in MeV or GeV and sometimes in fm^{-1}; lengths are usually quoted in fm, but sometimes in MeV^{-1} or GeV^{-1}. $1\ \mathrm{fm}^{-1} = 197.33$ MeV.

Table 1.1. Properties of quarks. Masses are model-dependent and hence have only qualitative meaning. Mass units are MeV. (*cf*. Gasser and Leutweyler, 1982).

flavor	symbol	charge	current mass	constituent mass
up	u	$+\frac{2}{3}$	6	330
down	d	$-\frac{1}{3}$	10	330
charmed	c	$+\frac{2}{3}$	1,350	2,000
strange	s	$-\frac{1}{3}$	199	520
top ?	t	$+\frac{2}{3}$	>50,000	>50,000
bottom	b	$-\frac{1}{3}$	5,000	5,000

The field operator ψ is taken to be a vector incorporating 6(?) flavor and 3 color, as well as the four Dirac, components. D_μ is the gauge-covariant derivative

$$D_\mu = \partial_\mu - i\tfrac{1}{2}g_s\lambda_c A^c_\mu \,. \tag{1.2}$$

The λ_c are the $3\times 3-1=8$ traceless Gell-Mann $SU(3)$ color matrices, constructed in analogy with the Pauli spin matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \frac{1}{\sqrt{3}} \,.$$

They satisfy the relationships

$$\operatorname{tr} \lambda_c \lambda_{c'} = 2\delta_{cc'}, \qquad \vec{\lambda}\cdot\vec{\lambda} \equiv \sum_c (\lambda_c)^2 = \tfrac{16}{3} I \tag{1.3}$$

Each of the $(\lambda_c)^2$ is diagonal. The commutators of the λ's are given by

$$[\lambda_c, \lambda_d] = 2\,i\, f_{cde}\lambda^e, \tag{1.4}$$

where the structure constants f_{cde} are completely antisymmetric in the three indices. The nonvanishing values are given by

$$\begin{aligned} &f_{123} = 1 \\ &f_{147} = f_{246} = f_{257} = f_{345} = -f_{156} = -f_{367} = \tfrac{1}{2} \\ &f_{458} = f_{678} = \sqrt{3}/2 \,. \end{aligned}$$

The upper and lower indices have the same meaning for both the λ's and the f's. g_s is the strong coupling constant. According to renormalization group arguments, it is not a fixed parameter, but rather must be treated as a running coupling constant dependent upon the characteristic length or momentum scale (Q) of the processes involved. The *effective* coupling constant $\alpha_s(Q^2) = g_s^2/4\pi$ is given in perturbation theory by

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\ln(Q^2/\Lambda_{QCD}^2)} , \tag{1.5}$$

where n_f is the number of flavors and Λ_{QCD} is the QCD momentum scale; experimentally, $\Lambda_{QCD} \approx$ 150-200 MeV. The formula displays asymptotic freedom, since $\alpha_s \rightarrow 0$ as $Q^2 \rightarrow \infty$. The formula strongly suggests that the number of flavors is limited to $n_f < 17$, which is comfortably within the current "best bet" of 6.

The antisymmetric gauge (gluon) field tensor is given by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^a{}_{bc} A^b_\mu A^c_\nu , \tag{1.6}$$

where the $f_{abc} = f^a{}_{bc}$, etc. are the SU(3) structure constants given above. The terms proportional to the $f's$ gives rise to the nonlinear, non-Abelian complications of the theory. From the Lagrangian follow the field equations

$$\partial^\mu F^c_{\mu\nu} = \tfrac{1}{2} g_s \bar{\psi} \gamma_\nu \lambda^c \psi - g_s f_e{}^{cd} A^\mu_d F^e_{\mu\nu}. \tag{1.7}$$

The time ($\nu = 0$) part of this equation is Gauss's law: The first term on the right hand side contains the quark color charge: the second term contains the color charge carried by the gauge fields, and again manifests the nonlinearity of the theory.

1.3 Why Modeling?

Because of the intractability of the QCD equations, and the current limitations of lattice gauge calculations, researchers have been lead to modeling. *Modeling* differs from *approximating* a theory in the following way: In the latter case one begins with what is assumed to be the basic theory and then makes mathematical approximations, which may or may not be justified, in order to render the problem tractable. In modeling, one proffers a physical system which is well-defined, possesses the essential properties of the basic theory, and is simpler to solve, but is generally not equivalent to the basic theory.

QCD modeling has a nice analogy with nuclear modeling. Even when one believed that one had a good theory of nuclear forces, the many body problem at first appeared to be intractable. The shell, optical, and collective models were invented to describe nuclear structure and reactions. When Brueckner theory was introduced, its first goals were to reproduce the empirically derived parameters of the phenomenological models. Similar examples can be found in atomic physics, solid state physics, superconductivity, etc. Of course modeling is also used when there is *no* basic theory.

The goal of modeling here is to bridge the gap between QCD and experiment in the realm of GeV energies and fm distances. At a minimum, the model should contain the following essential features of QCD: Absolute color confinement and asymptotic freedom, along with the elementary properties of the fundamental quark and gluon fields.

1.4 The MIT model

The prototype of bag models is the MIT bag model (Chodos, *et al.*, 1974; see also Bogoliubov, 1968). In its original form, it is elegantly simple. Quarks move freely within a cavity of radius R. The boundary condition on the quark wave function is identical to that obtained by assuming a scalar potential which is zero inside the cavity and infinite outside. There is an energy density B within the volume of the bag; B can also be regarded as the pressure of the vacuum on the bag. Furthermore, there is a chromo-dielectric function which is unity within the bag and zero outside.

Inside the bag, the quarks satisfy the Dirac wave equation

$$(-i\vec{\alpha} \cdot \vec{\nabla} + \beta m)\psi = \epsilon\psi \,, \tag{1.8}$$

where m is the flavor (current) mass matrix. The boundary condition on the quark wave function is

$$-i\vec{\gamma}\cdot\hat{n}\,\psi(R) = \psi(R)\,, \tag{1.9}$$

where $\hat{n}$ is the normal to the surface. For the case of a spherical bag, the solutions are characterized by the Dirac quantum number κ [see Eq. (3.11)] and for $m = 0$ can be written in the (unnormalized) form

$$\psi_\kappa = \begin{pmatrix} j_{-\kappa-1}(\omega r/R) \\ i\vec{\sigma}\cdot\hat{r}\, j_{-\kappa}(\omega r/R) \end{pmatrix} \mathcal{Y}^l_{jm}(\hat{r}) \qquad \kappa < 0\,. \tag{1.10a}$$

and

$$\psi_\kappa = \begin{pmatrix} j_{\kappa}(\omega r/R) \\ -i\vec{\sigma}\cdot\hat{r}\, j_{\kappa-1}(\omega r/R) \end{pmatrix} \mathcal{Y}^l_{jm}(\hat{r})\,. \qquad \kappa > 0 \tag{1.10b}$$

Here the $\mathcal{Y}^l_{jm}$ are the spherical harmonic two component spinors [see Eq. (3.10)], and the j_l are the spherical Bessel functions that are regular at the origin. The ω's are the dimensionless eigenvalues determined by the boundary condition, which yields the transcendental equations

$$j_{-\kappa-1}(\omega) = j_{-\kappa}(\omega) \qquad \kappa < 0 \tag{1.11a}$$

and

$$j_{\kappa}(\omega) = -j_{\kappa-1}(\omega)\,. \qquad \kappa > 0 \tag{1.11b}$$

The upper and lower components for the $\kappa = -1$ ($s_{1/2}$) state are displayed in Fig. 1.1.

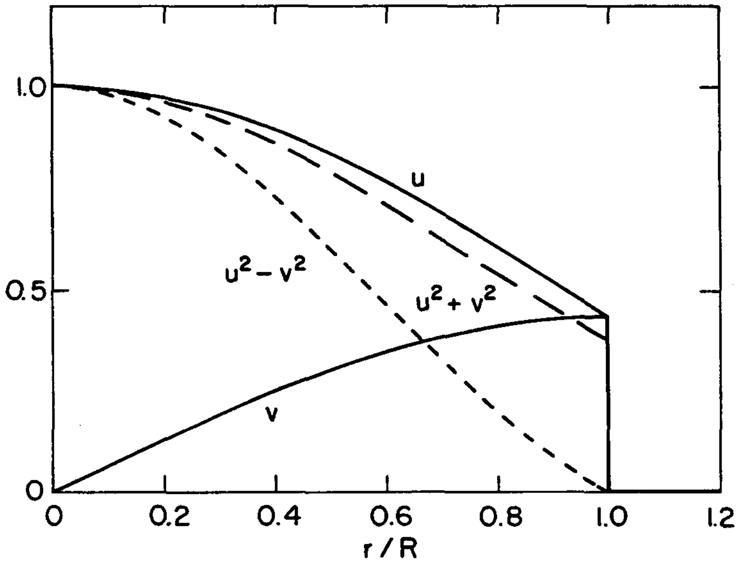

Figure 1.1. Upper (u) and lower (v) components for the lowest positive energy, $\kappa = -1$, state in the MIT bag. The density $u^2 + v^2$ and scalar density $u^2 - v^2$ are also shown. Compare Figs. 3.1 and 8.3.

The first few energy levels for zero mass quarks are

$$\kappa = -1\ (s_{1/2}) : \ \omega = 2.043,\ 5.396,\ 8.578,\ \cdots$$
$$\kappa = +1\ (p_{1/2}) : \ \omega = 3.812,\ 7.002,\ \cdots$$
$$\kappa = -2\ (p_{3/2}) : \ \omega = 3.204,\ 6.758,\ \cdots$$
$$\kappa = +2\ (d_{3/2}) : \ \omega = 5.123,\ 8.408,\ \cdots$$

Note the inverted spin-orbit coupling (with respect to the hydrogen atom): for the same l, the higher j lies lower. Thus the $p_{3/2}$ state lies lower than the $p_{1/2}$. This is characteristic of scalar potentials in contrast to vector potentials.

Adding the volume energy to the quark energy, one obtains for N_q quarks

$$E = \frac{N_q \omega_{-1}}{R} + \tfrac{4}{3}\pi R^3 B\,. \tag{1.12}$$

The minimum in E occurs at $R = R_0$ where

$$R_0^4 = \frac{N_q \omega_{-1}}{4\pi B} . \tag{1.13}$$

The bag constant B is the only parameter to this point. One can use (1.12) and (1.13) to obtain various relationships among E, R_0 and B. For example, one can relate the mean baryon (nucleon or delta, $N_q = 3$) mass $\overline{m}$ to the mean radius $\overline{R}$

$$E(\overline{R}) \equiv \overline{m} = \frac{4\,\omega_{-1}}{\overline{R}} = \frac{8.172}{\overline{R}} \tag{1.14}$$

and

$$B = \frac{3\,\overline{m}^4}{1024\,\pi\omega_{-1}^3} = 1.0936 \times 10^{-4}\overline{m}^4 . \tag{1.15}$$

With $\overline{m} = 1087$ MeV, (1.14) gives $\overline{R} = 1.48$ fm, and (1.15) gives $B = 19.9$ MeV/fm^3. Through first order in α_s, gluon corrections split the baryon masses and radii symmetrically about the mean values. Details of gluonic corrections are given in Chapter 6. For present purposes we can simply add to (1.12) a gluonic correction term proportional to α_s/R. One can then relate the change in bag radius to the change in bag energy by

$$\frac{\delta R}{\overline{R}} = \frac{\pm\frac{1}{2}\Delta m}{3\overline{m}} \qquad \text{for the } \begin{pmatrix} \Delta \\ N \end{pmatrix} , \tag{1.16}$$

where $\Delta m = m_\Delta - m_N = 297$ MeV. This gives a correction to the bag radius of $\pm 4.6\%$, bringing the nucleon bag radius to 1.41 fm.

In the work of DeGrand *et al.* (1975), a term representing the zero-point gluon energy in a cavity, the Casimir (1948) effect, was introduced phenomenologically as $-Z_0/R$. (It was also intended to incorporate other effects.) This is equivalent, for the baryon, to replacing ω_{-1} by $\omega_{-1} - \frac{1}{3}Z_0$ in Eqs. (1.12) - (1.15). Eq. (1.16) is not affected. With the value $Z_0 = 1.84$, which they obtained by fitting data, they found $R_N = 0.99$ fm and $B = 57.5$ MeV/fm^3.

The Casimir effect is both problematical and controversial. It certainly cannot be used without some form of cut-off regularization. This can be seen by noting that the energy of an empty bag,

$$-\frac{Z_0}{R} + \tfrac{4}{3}\pi R^3$$

with $Z_0 > 0$, is monotonic and unbounded from below. Thus a vacuum sprinkled with holes (or even one hole) would be unstable. The cut-off radius for a zero energy bag would be $(3Z_0/4\pi B)^{1/4}$. For the parameters quoted in the previous paragraph, this would yield a (minimum) radius of 1.1 fm, which is inconsistent with the hadron bag radii derived in the calculations. Furthermore, there are ambiguities in the derivation of the term, and even in the sign of the term. (Casimir's parallel capacitor is a conductor, whereas the MIT cavity is surrounded by a perfect dia-electric).

The quark density distribution drops approximately parabolically with radius to a value at the surface equal to 0.3800 of the central value, see Fig. 1.1. The rms quark radius is $0.7290\,R$ compared with $\sqrt{3/5}R = 0.7746\,R$ for a uniform distribution. That is to say, the bag radius is 6% *larger* than what is called the equivalent (charge) radius.

The nucleon magnetic moments are given by $\mu_n = -\frac{2}{3}\mu_p$ with $\mu_p = 2.20/2m$, where $1/2m$ is the magneton; the experimental value of μ_p is 2.7928456. The ratio of the axial and vector coupling constants is given in the model by $g_A/g_V = 1.09$ compared with 1.26 for experiment.

1.5 Evolution of the MIT model

Violation of the conservation of axial current (CAC) for systems of massless quarks has plagued bag models from the beginning. (In the presence of massless pions, CAC becomes chiral invariance.) In the MIT model, the axial current is finite in the radial direction at the inner edge of the bag boundary; the divergence of the axial current is infinite across the boundary. In order to remedy this problem, Chodos and Thorn (1975) and Vento, Rho, Nyman, Jun and Brown (1980) coupled an elementary pion field to the bag surface so as to render the axial current continuous. For pions of finite mass, this yielded partial conservation of the axial current (PCAC).

A highly successful program involving quark-pion bags was undertaken by Miller, Thomas and Thèberge (1980) under the name of the cloudy bag model (CBM).

A hybrid model which divides the space of a hadron into an interior MIT bag and an exterior topological Skyrme model has been dubbed the Cheshire cat model (Nadkarni, Nielsen and Zahed, 1985; Vepstas, Jackson and Goldhaber, 1984).

1.6 Why a soliton model?

Static, boundary condition bag models such as the MIT and CBM, have had considerable success in reproducing the spectra and properties of low-lying hadronic states involving light quarks. A difficulty which all such models encounter, however, has been the handling of the *dynamics* of the confinement mechanism (*cf.* Hasenfratz and Kuti, 1978). Soliton models have the important feature that confinement is effected by the intervention of a quantum mechanical scalar field. The effective Lagrangian contains the time derivatives of the fields so that a Hamiltonian can be constructed which contains the field and its conjugate momentum. Methods familiar from nuclear theory can therefore be used to construct fully quantal states of the system. In doing so, one can employ, for example, the coherent state (or, more generally, the single mode) approximation for the scalar field part of the state vector. This is related to the mean field approximation, but is quantal.

[A full treatment of the gluons as dynamical fields still leads to complications. Satisfaction of a gauge condition, for example as in QED, may eliminate a time derivative (Hasenfratz and Kuti, 1978); there are well known techniques for quantization in that case.]

The Lagrangian for the soliton model is the usual QCD Lagrangian supplemented by a non-linear scalar sigma field, which may be interpreted as representing the gluon condensate arising from the non-linear interactions of the color fields. Since the gluons are also represented in the Lagrangian, there is clearly double counting, in principle at least. This does not arise to the order of one gluon exchange, since the sigma field is color-singlet, and one gluon always carries color; two gluon exchange could involve two gluons in a color singlet state and hence could also be contained in the exchange of a sigma quantum. The Lagrangian also contains a color-dielectric function which is a function of the sigma field. The form of the dielectric function assures color confinement.

Parameters associated with the sigma field are adjusted (regularized) to yield physical results for calculated hadronic properties. If we could solve the model Lagrangian exactly, we would obtain the exact QCD results when the model parameters are adjusted to decouple the sigma field from the system. Since the parameters of the sigma field are to be readjusted at every level of the calculation to fit key data, the results of calculations should converge to the exact QCD values. Of course, no one has yet been clever enough to calculate hadronic properties exactly in QCD; so long as the calculations based on the soliton model remain at a relatively simple level, the model must be regarded as phenomenological.

Using a phenomenological form for the gluon propagator, Cahill and Roberts (1985) have proffered an interesting "derivation" of the soliton model.

We will see in Chapter 8 how the model can incorporate CAC (or chiral invariance).

1.7 The variegate world of solitons

Solitary waves were discovered by J. Scott Russell (1845) who observed stable, large amplitude water waves generated by a horse-drawn boat in a narrow channel. The nonlinear hydrodynamic equations admit solutions of finite amplitude waves which propagate with stable wave form, without dispersion. Zabusky and Kruskal (1965) coined the name *soliton* and the term has stuck and has been extended to a variety of nonlinear wave phenomena. In hadronic physics, it is used for structures built of Bosons.

One classification of solitons is according to "topological" and "nontopological" (*cf.* Lee, 1981). Topological solitons have at least two phases of the system which are degenerate in energy density, *i.e.* a degenerate vacuum. Thus the space can be divided into topologically distinct regions of low energy. To transform a region from one state to the other would take finite (or infinite) energy, leading to stability of the topology classically. Nontopological solitons acquire their stability as a result of the presence of some conserved charge, *e.g.*, baryon number for quarks. The Skyrme model (Skyrme, 1961; Witten, 1983) is a famous example of topological solitons.

A variety of models of QCD in the category of nontopological solitons deserve more attention than are reported here. These include the linear sigma model (Birse and Banerjee, 1984, 1985; Kahana, Ripka and Soni, 1984) and the very extensive work of Celenza and Shakin (1987) on a primarily linear version.