§2.4 Pseudorapidity Variable

To characterize the rapidity of a particle, it is necessary to measure two quantities of the particle, such as its energy and its longitudinal momentum. In many experiments, it is only possible to measure the angle of the detected particle relative to the beam axis. In that case, it is convenient to utilize this information by using the *pseudorapidity variable* η to characterize the detected particle. The pseudorapidity variable of a particle c is defined as

$$\eta = -\ln[\tan(\theta/2)]\,, \tag{2.24}$$

where θ is the angle between the particle momentum $\boldsymbol{p}$ and the beam axis. In terms of the momentum, the pseudorapidity variable can be written as

$$\eta = \frac{1}{2}\ln\left(\frac{|\boldsymbol{p}|+p_z}{|\boldsymbol{p}|-p_z}\right). \tag{2.25}$$

By comparing Eqs. (2.14) and (2.25), it is easy to see that the pseudorapidity variable coincides with the rapidity variable when the momentum is large, that is, when $|\boldsymbol{p}| \approx p_0$.

We consider the change of variables from $(y, \boldsymbol{p}_T)$ to $(\eta, \boldsymbol{p}_T)$. It is easy to express y as a function of η, and vice versa. From the definition of η, we have

$$e^{\eta} = \sqrt{\frac{|\boldsymbol{p}|+p_z}{|\boldsymbol{p}|-p_z}} \tag{2.26}$$

and

$$e^{-\eta} = \sqrt{\frac{|\boldsymbol{p}|-p_z}{|\boldsymbol{p}|+p_z}}\,. \tag{2.27}$$

Adding Eqs. (2.26) and (2.27), we obtain the relation

$$|\boldsymbol{p}| = p_T \cosh\eta\,,$$

where p_T is the magnitude of the transverse momentum:

$$p_T = \sqrt{\boldsymbol{p}^2 - p_z^2}\,.$$

Subtracting Eq. (2.27) from (2.26), we obtain

$$p_z = p_T \sinh\eta\,. \tag{2.28}$$

Using these results, we can express the rapidity variable y in terms of the pseudorapidity variable η as

$$y = \frac{1}{2} \ln \left[\frac{\sqrt{p_T^2 \cosh^2 \eta + m^2} + p_T \sinh \eta}{\sqrt{p_T^2 \cosh^2 \eta + m^2} - p_T \sinh \eta} \right], \tag{2.29}$$

where m is the rest mass of the particle. Conversely, the pseudorapidity variable η can be expressed in terms of the rapidity variable y by

$$\eta = \frac{1}{2} \ln \left[\frac{\sqrt{m_T^2 \cosh^2 y - m^2} + m_T \sinh y}{\sqrt{m_T^2 \cosh^2 y - m^2} - m_T \sinh y} \right]. \tag{2.30}$$

If the particles have a distribution $dN/dyd\boldsymbol{p}_T$ in terms of the rapidity variable y, then the distribution in the pseudorapidity variable η is

$$\frac{dN}{d\eta d\boldsymbol{p}_T} = \sqrt{1 - \frac{m^2}{m_T^2 \cosh^2 y}} \frac{dN}{dyd\boldsymbol{p}_T}. \tag{2.31}$$

In many experiments, only the pseudorapidity variable of the detected particles is measured to give $dN/d\eta$, which is the integral of $dN/d\eta d\boldsymbol{p}_T$ with respect to the transverse momentum. One can compare this quantity with dN/dy, which is the integral of $dN/dyd\boldsymbol{p}_T$ with respect to the transverse momentum. From Eq. (2.31), we can infer that in the region of y much greater than zero, $dN/d\eta$ and dN/dy are approximately the same, but in the region of y close to zero, there is a small depression of the $dN/d\eta$ distribution relative to dN/dy due to the above transformation (2.31). In experiments at high energies where dN/dy has a plateau shape, this transformation gives a small dip in $dN/d\eta$ around $\eta \approx 0$.

The transformation (2.31) reveals the difference in the maximum magnitude of the pseudorapidity distribution for $dN/d\eta$, whether η is measured in the laboratory frame or in the center-of-mass frame. In the center-of-mass frame, the peak of the distribution is located around $y \approx \eta \approx 0$, and the peak value of $dN/d\eta$ is smaller than the peak value of dN/dy by approximately the factor $(1-m^2/<m_T^2>)^{1/2}$. In the laboratory frame, the peak of the distribution is located around half of the beam rapidity $\eta \approx y_b/2$ for which the factor $[1 - m^2/<m_T^2> \cosh^2(y_b/2)]^{1/2}$ is about unity. The peak value of

$dN/d\eta$ is approximately equal to the peak value of dN/dy. Because the shape of the rapidity distribution dN/dy does not change when one goes from the center-of-mass to the laboratory frame, the peak value of the pseudorapidity distribution in the center-of-mass frame is lower than the peak value of the pseudorapidity distribution in the laboratory frame.

§Reference for Chapter 2

1. W. Pauli, *Theory of Relativity*, Pergamon Press, N.Y., 1958, p. 25.