FOREWORD

In January 1956, in an address to the *All Union (USSR) Conference on Functional Analysis and Its Applications*, I. M. Gelfand said: "Two areas, in my opinion, will exert the strongest influence of all on the future course of development of functional analysis. The first of these is hydrodynamics (the problem of flow of a viscous fluid, the theory of a compressible gas, the theory of turbulence). The second area is theoretical physics, more precisely, the quantum theory of fields and the theory of elementary particles. This region of physics, it is true, itself stands at a crossroads and it is not clear what the nature of its development will be, but, however it develops, one thing is clear: it and functional analysis are to be fellow-travellers."

The three and a half decades which have since elapsed have seen mathematical bones put into the flesh of the second part of this pronouncement.

First, several mathematically precise definitions of what should constitute a quantum theory of fields were given and a general theory of quantized fields developed. The initial stages of this development took roughly a decade (from the mid-1950s to the mid-1960s), but work on the general theory continues to this day. The creation of this theory almost immediately paid off in a strong and fruitful interaction with the theory of von Neumann algebras, a core area in non-commutative functional analysis.

The next decade (from the mid-1960s to the mid-1970s) saw the initial stages of development in what has come to be called constructive quantum field theory: the mathematical construction of fields that are solutions of specific Lagrangian quantum field theories. The first examples treated (ϕ_4^2, $P(\phi)_2$, Y_2, ϕ_3^4 in the shorthand of the subject) were deliberately chosen so as to eliminate all inessential complications. They were therefore of somewhat limited physical interest. However, these examples did answer the question of whether non-trivial theories of interacting fields exist. The answer was yes, at least in spacetimes of dimension two and three.

The next logical step might appear to be the construction of theories in four dimensional spacetime, but that is not in fact what happened; the history in the decade and a half leading up to the present have been more complicated. On the one hand, it has been shown that the methods used so successfully in spacetime dimensions two and three to construct the theories ϕ_2^4 and ϕ_3^4 of a self-interacting scalar field, ϕ, fail for ϕ_ν^4, $\nu \geq 4$. On the other hand, quantum field theory in two and three dimensions has turned out to be applicable to a rich variety of physical phenomena.

The papers of Jürg Fröhlich collected here contain many important original contributions to the developments of the last decade and a half, but what may make them even more useful to the reader is the remarkable overview they provide. It would be fruitless and redundant to spell this out in detail here, since the author himself has provided an admirable commentary in the following introduction. It remains only to wish the reader many happy hours reading this book.

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