Contents

Preface vii

I Preliminaries 1

1 Introduction 3
1.1 History . . . 3
1.2 Four-Fermi interaction . . . 5
1.2.1 Modern form of four-Fermi interaction . . . 5
1.2.2 Problems with the four-Fermi interaction . . . 8
1.3 Symmetries and forces . . . 9
1.3.1 Global symmetries . . . 9
1.3.2 Local symmetries . . . 10
1.3.3 Spontaneous breaking of symmetries . . . 12
1.4 Renormalizability and anomalies . . . 16
References . . . 17

2 The standard model and the neutrino 19
2.1 Gauge interactions in the Standard model . . . 20
2.2 Neutral current interactions of neutrinos . . . 24
2.3 Neutrino scattering in the Standard model . . . 26
2.3.1 $\nu_e e$ and $\widehat{\nu}_e e$ scattering . . . 26
2.3.2 $\nu_\mu e$ and $\widehat{\nu}_\mu e$ scattering . . . 28
2.3.3 $\nu_e N$ and $\widehat{\nu}_e N$ scattering . . . 29
2.3.4 Neutrino pair production . . . 30
2.4 Neutrino mass in the standard model . . . 31
References . . . 33

3 Massive neutrinos **35**
3.1 Motivations for neutrino mass 35
3.1.1 Theoretical motivations in Particle physics 36
3.1.2 Motivations from Astrophysics and Cosmology . . 37
3.2 Questions related to neutrino mass 40
3.3 Tests of neutrino mass 41
3.3.1 Kinematic tests 41
3.3.2 Exclusive tests 42
References 44

4 Dirac vs. Majorana masses **46**
4.1 Two-component spinor field 46
4.2 Mathematical definition of a Majorana field 49
4.3 Different representations of Dirac matrices 52
4.3.1 Dirac representation 54
4.3.2 Majorana representation 54
4.3.3 Other representations 55
4.4 Majorana neutrinos and discrete symmetries of space-time 56
4.4.1 Properties under $\mathcal{C}$ 56
4.4.2 Properties under $\mathcal{CP}$ 58
4.4.3 Properties under $\mathcal{CPT}$ 59
4.5 The Majorana basis of mass terms 60
4.6 Diagonalizaton of fermion mass matrices 64
References 66

II Models of neutrino mass **67**

5 Neutrino mass in $SU(2)_L \times U(1)_Y$ models **69**
5.1 Models with enlarged fermion sector 69
5.1.1 A simple model with Dirac neutrinos 70
5.1.2 Neutrino mixing 71
5.1.3 Shortcomings of the model 71
5.1.4 The complete model with right handed neutrinos . 72
5.2 Models with expanded Higgs sector 76

5.2.1 Adding a triplet Δ 77
5.2.2 Model with a singly charged singlet 79
5.2.3 Model with doubly charged singlet 82
5.3 The method of flavor diagrams 83
5.4 Models with spontaneous $B - L$ violation 86
5.4.1 Constraints on Majoron models 86
5.4.2 Majoron in the model with right-handed neutrinos 89
5.4.3 Majorons in models with extended Higgs sector . . 91
References 94

6 Neutrino mass in Left-Right symmetric models 96
6.1 The gauge sector 97
6.1.1 Symmetry breaking 97
6.1.2 Constraints on the masses of the gauge bosons . . 102
6.2 Majorana neutrinos 105
6.2.1 The see-saw mechanism 105
6.2.2 Constraints on the eV-keV-MeV spectrum 107
6.3 Physics involving right-handed neutrinos 110
6.3.1 Flavor changing neutral currents 111
6.3.2 Decay of the right-handed neutrinos 112
6.4 Naturalness of the see-saw formula 115
6.5 Dirac neutrinos 117
References 120

7 Neutrino mass in Grand Unified models 123
7.1 SU(5) 124
7.2 Neutrino masses in $SU(5)$ model 126
7.3 SO(10) 128
7.4 Neutrino mass in $SO(10)$ models 132
7.5 Neutrino masses in E_6 139
References 142

III Phenomenology 145

8 Kinematic tests of the neutrino mass **147**
8.1 Beta decay and the mass of the ν_e 147
8.2 Pion decay and the mass of the ν_μ 151
8.3 Taon decay and the mass of the ν_τ 152
8.4 Other processes 153
8.4.1 Electron capture: mass of the ν_e 153
8.4.2 Kaon decay: mass of the ν_τ 153
References 154

9 Neutrino oscillations **155**
9.1 Neutrino oscillations in vacuum 155
9.1.1 Theory of neutrino oscillations 156
9.1.2 Experimental searches of neutrino oscillations . . . 158
9.1.3 Atmospheric neutrinos 163
9.2 Oscillation with unstable neutrinos 163
9.3 Neutrino oscillations in matter 165
References 171

10 Electromagnetic properties of neutrinos **173**
10.1 Electromagnetic form factors of a neutrino 174
10.1.1 Form factors of a Dirac neutrino 175
10.1.2 Form factors of a Majorana neutrino 176
10.1.3 Form factors for a Weyl neutrino 178
10.2 Kinematics of radiative decays 179
10.3 Model calculations 181
10.3.1 $SU(2)_L \times U(1)_Y$ model with Dirac neutrinos . . . 181
10.3.2 $SU(2)_L \times U(1)_Y$ models with Majorana neutrinos 186
10.3.3 Left-right symmetric model 189
10.4 Large magnetic moment and small neutrino mass 193
References 196

11 Double beta decay **198**
11.1 Introduction 198
11.2 Kinematical properties 200
11.3 Neutrinoless double beta decay in $SU(2)_L \times U(1)_Y$ models 204

11.3.1 Light Majorana neutrino exchange 204
11.3.2 Heavy Majorana neutrino exchange 207
11.3.3 Exchange of doubly charged Higgs boson 208
11.4 Neutrinoless double beta decay in Left-Right models . . . 209
11.4.1 Light neutrino exchange 209
11.4.2 Heavy Majorana neutrino exchange 210
11.4.3 Left-right mixing contribution 211
11.4.4 Higgs exchange contribution 212
11.5 Majoron emission in $\beta\beta_{0\nu}$ decay 212
11.6 Neutrino mass and $\beta\beta_{0\nu}$ decay 214
References 216

12 Related processes **218**
12.1 Lepton flavor changing processes 218
12.1.1 Radiative decays of muon and taon 218
12.1.2 Decays of μ and τ into charged leptons 221
12.1.3 Muonium-antimuonium transition 224
12.1.4 Semi-leptonic processes 227
12.2 CP-violation in the leptonic sector 229
12.2.1 CP-violating phases in the fermion mass matrix . . 229
12.2.2 Electric dipole moment of the electron 231
References 234

IV Astrophysics and Cosmology **237**

13 Solar neutrinos **239**
13.1 Solar neutrinos 240
13.1.1 Source of neutrinos in the sun 240
13.1.2 Detection of solar neutrinos 242
13.2 The solar neutrino puzzle 244
13.3 Solar neutrino puzzle and neutrino properties 247
13.3.1 Neutrino oscillation 247
13.3.2 Neutrino decay 253
13.3.3 Magnetic moment of the neutrino 254

13.4 Implications for Particle Physics 258
References 259

14 Neutrinos from Supernovae **262**
14.1 Qualitative picture of supernova collapse 262
14.2 Flux of supernova neutrinos 264
14.3 Neutrino properties implied by SN1987A observations . . 268
14.3.1 Neutrino mass 269
14.3.2 Neutrino lifetime 270
14.3.3 Magnetic moment of the neutrino 270
14.3.4 Electric charge of neutrino 273
14.3.5 Strength of right-handed weak interactions 274
14.3.6 Radiative decay of neutrinos 275
14.3.7 Test of weak equivalence principle for neutrinos . . 277
References 278

15 Neutrino cosmology **280**
15.1 The Big Bang model 280
15.2 Nucleosynthesis and the number of neutrino species . . . 284
15.3 Constraints on stable neutrino masses 288
15.3.1 Bounds on light neutrino masses 289
15.3.2 Heavy stable neutrinos 290
15.4 Constraints on heavy unstable neutrinos 293
15.5 Limits for radiative neutrino decays 295
15.6 Limits on right handed interactions 299
15.7 Neutrinos and dark matter in the universe 301
15.7.1 Galactic halos and neutrinos 301
15.7.2 Galaxy formation and neutrinos 305
References 307

A Exercises **309**

B Addenda **314**

Index **315**