

The *cyclotron radius*  $\rho$  is just the radius of the particle's orbit. Eq. (1.22) now becomes

$$q\vec{v} \times \vec{B} = \gamma m \vec{\omega} \times \frac{d\vec{\rho}}{dt} = \gamma m \vec{\omega} \times \vec{v}, \quad (1.23)$$

or for a particle moving in a plane perpendicular to  $\vec{B}$ ,

$$evB = \gamma m \omega v = \gamma m \frac{v^2}{\rho}, \quad (1.24)$$

i. e., the Lorentz force is the centripetal force which keeps the particle of charge  $q$  and mass  $m$  on a circular orbit. Dividing Eq. (1.24) by  $v/\rho$ , we get a relation for the momentum in terms of the orbit radius, magnetic field and charge of the particle:

$$p = \beta \gamma m c = eB\rho. \quad (1.25)$$

For a particle with same charge as the electron, it is useful to remember

$$p(\text{GeV}/c) \simeq 0.3B(\text{T})\rho(\text{m}). \quad (1.26)$$

Another popular formula is the one for the angular velocity or cyclotron frequency:

$$\omega = \frac{eB}{\gamma m} \quad (1.27)$$

It may also be useful to remember the Lorentz transformations of the electromagnetic field:

$$\vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \vec{v} \times \vec{B}_{\perp}), \quad (1.28)$$

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel}, \quad (1.29)$$

$$\vec{B}'_{\perp} = \gamma(\vec{B}_{\perp} - \vec{v} \times \vec{E}_{\perp}), \quad \text{and} \quad (1.30)$$

$$\vec{B}'_{\parallel} = \vec{B}_{\parallel}, \quad (1.31)$$

where the  $\parallel$  designates the component of the field parallel to the boost velocity  $\vec{v}$ , and  $\perp$  indicates the component perpendicular to the boost.

#### 1.4 Linear accelerators with oscillating electric fields

Since it is very difficult to produce dc voltages more than a few million volts, it was necessary to find a new method for acceleration to energies beyond a few MeV. In 1928 Wideröe proposed an accelerating structure using a series of cylindrical

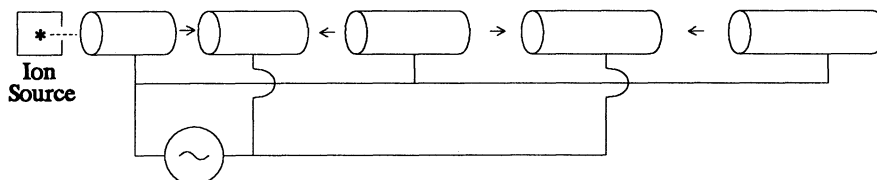


Figure. 1.5 A schematic of the Wideröe linac structure. The arrows indicate the directions of the accelerating electric field at a given instant of time.

tubes, called drift tubes, which were alternately connected to a high frequency oscillator, as shown in Fig. 1.5. Charged particles from the source are accelerated in the gaps between tubes. They then drift in the field free region inside the tube. While the particles are inside the tube the direction of the field is reversed so that when the particles reach the next gap, they again see an accelerating electric field. If a constant frequency generator is used, the tubes must increase in length as the particle velocity increases, so that the particles will always arrive at the next gap with the correct phase of the accelerating voltage in the gap. Particles will only leave the source when the voltage in the first gap has the correct sign to accelerate them, thus the accelerated beam will have a pulsed structure to it.

A cell is usually defined as the region from the midplane of one drift tube to the next. (Sometimes it is convenient to offset this slightly, but the length of the cell remains the same.) The Wideröe structure is called a  $\beta\lambda/2$  or  $\pi$ -mode structure, since the electric field configuration repeats every two cells. The product  $\beta\lambda$  is the distance that the particle travels during one rf cycle. As the particles' velocities increase, the cell lengths must increase. Bunches of particles cannot be accelerated in every gap during a half cycle, but must be spaced with a free gap between every pair of bunches.

The Alvarez structure, shown in Fig. 1.6, is a  $\beta\lambda$  or  $2\pi$ -mode structure and can accelerate particles simultaneously in each gap. Since this is a  $2\pi$ -mode structure, the rods connecting the tank and drift tubes are only necessary for support. In this structure the charges oscillate between the ends of the drift tubes. Unlike the Wideröe structure with the charges actually traveling from one tube to the next by passing through the rf generator, the Alvarez structure is a resonant structure which is inductively coupled through a transformer consisting of a one turn primary inserted through the wall of the resonant tank containing the drift tubes.

Clearly, as the particles become more relativistic, the length of a cell increases.

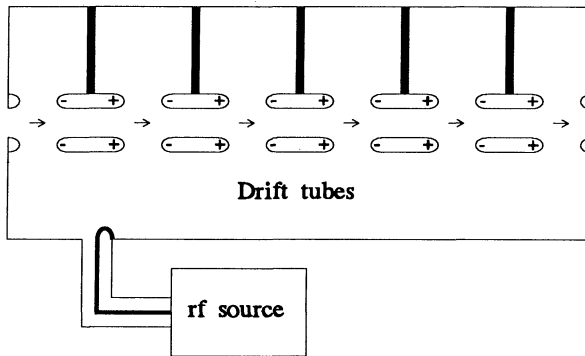


Figure. 1.6 The Alvarez drift tube structure. This is a  $2\pi$ -mode structure with the field pattern repeating in every cell. The arrows indicate the direction of the electric field at one instant.

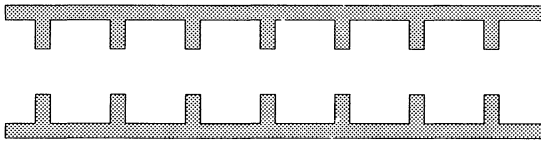


Figure. 1.7 A traveling wave disk and washer structure.

To counteract this requires either a much longer cell, or a source of higher frequency. Conventional triode and tetrode tubes were unable to operate at high frequencies in the microwave regime. Another type of tube called a klystron<sup>5</sup> can produce very high power at frequencies from a few hundred megahertz to several tens of gigahertz. The klystron is really more like a small linear accelerator than an electron vacuum tube. It uses a driven rf cavity to modulate a dc beam by varying the velocity of the particles with respect to the time of passage through the cavity. The particles drift for some distance and accumulate into bunches which appear as a pulsed current at a second resonant output cavity. The output power can be coupled by an inductive loop to a waveguide that pipes the power to an accelerating structure.

At about the same time as the invention of the klystron, it was realized that a traveling wave could be used to accelerate relativistic particles. A cylindrical waveguide propagates waves with phase velocities greater than the speed of light. Since the charged particles must be traveling at less than the speed of light, they will not obtain any net acceleration, because they cannot keep in phase with the wave. If the waveguide is loaded by corrugating its walls, so that the induced charges have

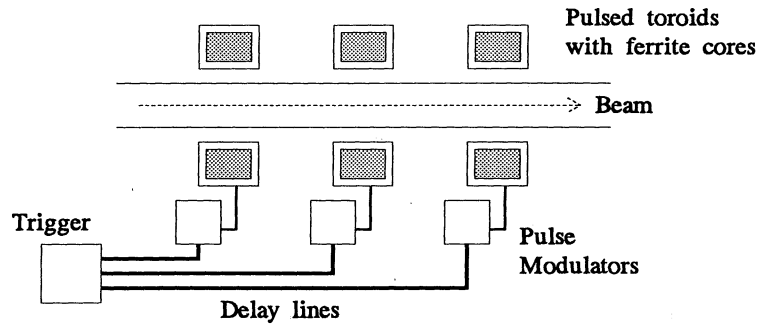


Figure. 1.8 An induction linac. This operates in the same manner as a transformer. The primary turns are toroidal pulsed electromagnets, and the secondary is the beam.

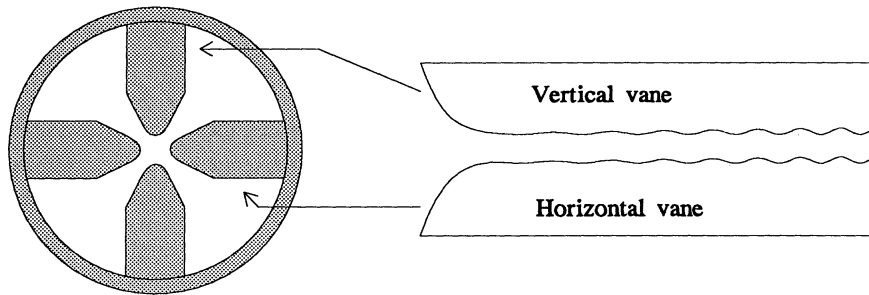


Figure. 1.9 A cross section of an RFQ structure. The longitudinal profile of adjacent vanes is shown to the right.

a longer path length (as in Fig. 1.7), the phase velocity of the wave can be slowed down to a usable value (or even slower.) The particles may then “surf” along the wave with a phase yielding an accelerating force. This type of structure is called a *traveling wave structure*.

A *standing wave* structure is a structure which has two traveling waves moving in opposite directions. This type of structure is necessary for accelerating oppositely charged particles ( $e^+e^-$  or  $p\bar{p}$ ) in opposite directions in the same accelerator.

Another type of linac is the induction linac (see Fig. 1.8.) This linac uses a series of toroidal electromagnets coaxially placed along the beam axis. By successively pulsing each magnet, large peak values of emf can be produced which will accelerate the beam. Typical currents of several kiloamperes can be achieved.

More recently, the radio frequency quadrupole (RFQ) has been developed

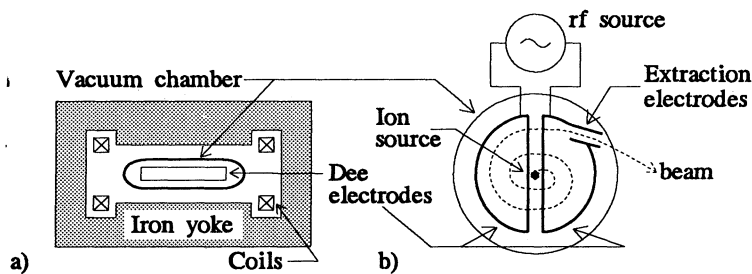


Figure. 1.10 a) Side view of a cyclotron. b) Top view of a cyclotron, with “D” electrodes shown inside the vacuum chamber.

for preliminary acceleration of protons and heavier ions. This uses four parallel electrodes around the beam axis as shown in Fig. 1.9. It is a resonant structure with adjacent electrodes having opposite charges. From the end, they look like an electric quadrupole. This arrangement of electric fields focuses the beam in one plane and defocuses the beam in the other plane. Since the electric field oscillates, a net focusing effect can be obtained. If the electrodes are scalloped with a curve somewhat like a sinusoid, and with the curves of the adjacent electrodes differing by  $180^\circ$  in phase, then a component of longitudinal field will be produced, which may be used to accelerate the particles.

### 1.5 Circular machines

The cyclotron is the first example of a circular machine.<sup>6,7</sup> A homogeneous magnetic field, supplied by an H-shaped magnet, as in Fig. 1.10a, bends back the particles to the same rf gap between the two D-shaped electrodes shown in Fig. 1.10b, twice each period of the radio frequency oscillation. If the rf is set equal to the cyclotron frequency (a resonance condition) given by Eq. (1.27) with  $\gamma = 1$  (N.R. ions or protons), the particles will continue to pass near the peak of the rf voltage twice per turn, gaining kinetic energy, and then increasing the radius of their orbits by Eq. (1.25), till reaching some extraction device.

Usually  $\Delta E \leq 200$  keV/turn, then for  $W_{\max} \simeq 20\text{--}25$  MeV, one can infer that some 100 to 125 turns suffice to achieve the wanted acceleration. The required frequency can be calculated from

$$\frac{\nu_{\text{rf}}}{B} = \frac{e}{2\pi m} \simeq 15 \text{ MHz} \cdot \text{T}^{-1}, \text{ for protons.} \quad (1.32)$$

Typical beam currents would be about 1mA.