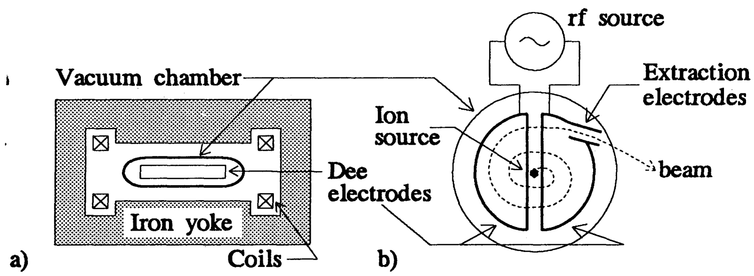

Figure. 1.10 a) Side view of a cyclotron. b) Top view of a cyclotron, with "D" electrodes shown inside the vacuum chamber.

for preliminary acceleration of protons and heavier ions. This uses four parallel electrodes around the beam axis as shown in Fig. 1.9. It is a resonant structure with adjacent electrodes having opposite charges. From the end, they look like an electric quadrupole. This arrangement of electric fields focuses the beam in one plane and defocuses the beam in the other plane. Since the electric field oscillates, a net focusing effect can be obtained. If the electrodes are scalloped with a curve somewhat like a sinusoid, and with the curves of the adjacent electrodes differing by 180° in phase, then a component of longitudinal field will be produced, which may be used to accelerate the particles.

1.5 Circular machines

The cyclotron is the first example of a circular machine.[6,7] A homogeneous magnetic field, supplied by an H-shaped magnet, as in Fig. 1.10a, bends back the particles to the same rf gap between the two D-shaped electrodes shown in Fig. 1.10b, twice each period of the radio frequency oscillation. If the rf is set equal to the cyclotron frequency (a resonance condition) given by Eq. (1.27) with $\gamma = 1$ (N.R. ions or protons), the particles will continue to pass near the peak of the rf voltage twice per turn, gaining kinetic energy, and then increasing the radius of their orbits by Eq. (1.25), till reaching some extraction device.

Usually $\Delta E \leq 200\,\text{keV/turn}$, then for $W_{\max} \simeq 20$—$25\,\text{MeV}$, one can infer that some 100 to 125 turns suffice to achieve the wanted acceleration. The required frequency can be calculated from

$$\frac{\nu_{\text{rf}}}{B} = \frac{e}{2\pi m} \simeq 15\,\text{MHz} \cdot \text{T}^{-1}, \text{ for protons.} \tag{1.32}$$

Typical beam currents would be about 1mA.

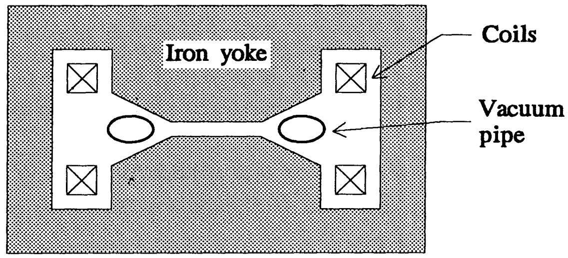

Figure. 1.11 Side cross section of a betatron.

However, Eq. (1.27) contains the Lorentz factor γ, which, as soon as the kinetic energy is significantly increased, begins to deviate from one, causing a decrease of the radio-frequency. A remedy to this consists of just varying the rf according to a $1/\gamma$ law; this means that only particles in phase with this varying rf can be accelerated. That is, a synchronous acceleration principle[8,9] is now required, and only one bunch of particles will reach the final kinetic energy (600—700 MeV for protons), due to technical limitations on magnet dimensions and rf modulation. The current is of the order of a few microamperes, but still a rather small number (a few thousand) of revolutions are required to accomplish the full cycle of acceleration.

This attention paid to the number of revolutions, run by particles from rest to their goal-energy, is dictated by the need of having orbits with limited dimensions. In fact, since particles leave the ion source with nonzero angles and energy spread to form a beam of nonzero size, a very high number of turns would imply a broadening of the beam, unless some focusing mechanism plays a role.

This problem first arose for the betatron,[10] a rather different machine based on acceleration of electrons by induction. The working principle of this accelerator will be illustrated briefly: the magnetic flux, crossing a circle of radius R, is $\phi = \pi R^2 \bar{B}$, where $\bar{B}$ is some average magnetic induction. If ϕ varies with time, an electron orbiting at this radius will experience a force,

$$F = -eE \simeq (-e)\frac{V}{2\pi R} = \frac{(-e)}{2\pi R}(-\frac{d\phi}{dt}) = \frac{1}{2}eR\frac{d\bar{B}}{dt}. \tag{1.33}$$

Newton's second law of dynamics gives

$$F = \frac{dp}{dt} = eR\frac{dB_g}{dt}, \tag{1.34}$$

having considered Eq. (1.25) with the field B_g in the magnet gap (see Fig. 1.11). From both Eqs. (1.33, and 1.34), it is easy to infer the 2–to–1 rule or Wideröe condition,[11]

$$B_g = \frac{1}{2}\bar{B}, \tag{1.35}$$

typical of betatrons, which gives rise to the rather cumbersome structure of these machines.

Working out a few realistic numbers, one obtains $W_{\text{max}} \simeq 50\,MeV$, which for $B_g = 0.5\,\text{T}$, gives $R = 33\,cm$ and $\bar{\tau}_{\text{rev}} = (2\pi R)/\bar{v} \simeq 14\,\text{ns}$, since $\bar{v} \simeq c/2$. For $\Delta t_{\text{cycle}} = (1/4 f_{\text{main}}) = 5\,\text{ms}$, we get

$$N_{\text{rev}} = \frac{\Delta t_{\text{cycle}}}{\bar{\tau}} \simeq 357,000 \text{ turns.} \tag{1.36}$$

Now the problem of focusing the circulating beam, mildly tackled in cyclotrons, becomes of primary importance. In the next chapter, we will derive the trajectory equations for a charged particle traveling through various shaped magnetic fields. Also, we will see that several effects will cause focusing in the dimensions transverse to the beam.

1.6 Momentum compaction and the synchronous particle

If a particle of fixed energy moves along a closed trajectory, the integral of the curvature, $1/\rho$, around the closed path must be

$$\oint \frac{ds}{\rho} = 2\pi. \tag{1.37}$$

Since $1/\rho = qB_\perp/p$, this means that the momentum

$$p = \frac{q}{2\pi} \oint B_\perp\, ds. \tag{1.38}$$

Leaving the magnetic guide field unchanged, consider another particle also with a closed trajectory but a slightly different momentum. This second particle's closed orbit must differ from the path of the first particle. By varying the momentum, the path length, $L = \oint ds$, of the closed trajectory can be varied. The fractional deviation of this path length divided by the fractional deviation of the momentum is frequently called *momentum compaction*

$$\alpha_p = \frac{dL}{L}\Big/\frac{dp}{p} = \frac{p}{L}\frac{dL}{dp}. \tag{1.39}$$

The name momentum compaction (see Appendix A) is reviled by some authors since an increase in momentum implies a lengthening of the orbit if $\alpha_p > 0$, i. e., a dilation rather than a compaction.