PREFACE

The aim of this book is to provide both a *tutorial* and a state-of-the-art *survey* on bifurcation and chaos, using Chua's circuit as the vehicle of discourse. While there have been a multitude of papers and books on the subject of chaos, most of these are concerned with the computer simulations of *hypothetical* nonlinear maps or differential equations, or with purely mathematical analysis. Very few publications are concerned with real experiments because, until recently, most experimental works have been published by physicists on fluid dynamics, plasmas, and turbulence where realistic yet tractable mathematical models are always under evolution. In other words, most publications necessarily address only one approach to chaos, numerical, mathematical, or experimental. Relatively few cover two out of the three approaches. In this regard, Chua's circuit stands out as the *only* real physical chaotic system which has been investigated in-depth using all three approaches, and in which the results predicted by the computer model coincided almost exactly with experimental measurements. It is also the only autonomous system of ordinary differential equations which has been proven mathematically to be chaotic in the sense of Shil'nikov's theorem.

In contrast, the classic Van der Pol equation represents only a rather poor model of the vacuum tube oscillator studied by Van der Pol in the twenties, which does not have a simple circuit realization using currently available components (e.g. diode, transistors, op amps, etc.). This fact is unfortunate because much of the published mathematical and numerical results on the Van der Pol equation cannot be verified experimentally, let alone exploited as a practical generator of chaos.

From the practical engineers' point of view, Chua's circuit is the chaotic circuit of choice because it can be built within a few minutes using off-the-shelf components. Its only nonlinear element can be implemented by either a dual op amp package and six resistors, or by an IC chip. In fact, the entire Chua's circuit has recently been fabricated in a CMOS monolithic chip.

Since Chua's circuit is endowed with an unusually rich repertoire of nonlinear dynamical phenomena, including all of the standard bifurcations and routes to chaos, it has become a *universal paradigm for chaos.* Because chaos is an ubiquitous phenomenon that cuts across all disciplines, thousands of papers involving hundreds of systems, both physical and hypothetical, have been published in dozens of journals. For a novice embarking on a serious research on chaos, it is often a time-consuming and frustrating experience to comb through the vast literature on chaos, to decipher the morass of unfamiliar terminologies and jargons, to filter out the main results, and then to unify them into a somewhat coherent body of knowledge. Because of its simplicity, Chua's circuit has become a standard primer on chaos for the uninitiated. By studying a single physical system experimentally, numerically and/or mathematically, one can learn in a relatively short time, all of the relevant aspects of chaos. For the advanced students of chaos, Chua's circuit offers a conceptual basis and a framework for new discoveries, especially in controlling, synchronizing, and harnessing chaos for novel applications. Its role in understanding the concepts and in inspiring the applications of nonlinear phenomenon and chaotic oscillations can be compared with the role that the L-C-R circuit has played traditionally in the existence, generation and applications of periodic oscillations. Though it is not possible to establish many parallels at this time, one can say that, in any application where an inexpensive, compact, and robust source of real chaotic signals is needed, Chua's circuit is the candidate of choice. By coupling many Chua's circuits into a 1-dimensional array, a ring, a 2-dimensional array, or a 3-dimensional lattice, many exciting new phenomena, including autowaves, nonlinear waves, complex patterns, self-organizations, complexities, etc., are waiting to be discovered, analyzed, explained, and exploited.

Altogether, this book contains 56 papers, with authors coming from 22 countries: Australia, Brazil, Chile, China, Czechoslovakia, Denmark, Egypt, France, Germany, Hungary, India, Italy, Japan, Poland, Russia, Spain, Switzerland, Turkey, Ukraine, United Kingdom, USA, and Yugoslavia. Most of these papers were reprinted from two recent Special Issues on "Chua's Circuit: A Paradigm for Chaos", vol. 3, no. 1 (March) and vol. 3, no. 2 (June) 1993, of the *Journal of Circuits, Systems, and Computers.*

In addition to several papers by Professor Chua and his co-workers, we are especially honored by contributions from two of the world's most renowned authorities on bifurcation and chaos; namely, Professor A. N. Sharkovsky from the Ukrainian Academy of Science and Professor L. P. Shil'nikov from the Research Institute of Applied Mathematics in Nishny Novgorod, Russia.

Rather than giving a synopsis to each paper in this book, we will briefly comment on some of the salient aspects of Chua's circuit that were addressed in these papers. In particular, there are tutorial papers written for beginners, including a pictorial guide on various periodic orbits, homoclinic orbits, and heteroclinic orbits, as well as a gallery of 48 colorful strange attractors associated with the canonical Chua's circuit. There are papers on computational techniques and tools including time series reconstruction, Lyapunov exponents, and a novel approach which exploits the number crunching power of a digital signal processor to achieve real-time simulation of the canonical Chua's circuit, and whose digital output can be converted into a real signal via a digital-to-analog converter, therefore providing a practical source of chaotic signals, each corresponding to a different strange attractor. There are papers on Chua's 1-D map which demonstrate its applications as an indispensable tool for extracting unstable periodic, homoclinic, and heteroclinic orbits from Chua's circuit. The reader should note papers that have presented new phenomena (e.g. stochastic resonance), identified geometric invariants, proved self-similarity, and universality, etc., in Chua's circuit. Adding to the diversity there are deep mathematical papers which characterize various models of Chua's attractors in a geometric setting, and in Banach spaces. There are papers which investigate the localization of attractors, and the dynamic range and basin of bifurcations. There are papers which developed special techniques (e.g. slow-fast dynamics, piecewise-linear techniques) that exploit the properties of Chua's equations.

There are papers which generalize Chua's circuit in various directions. In particular, there is an experimental high-frequency version of Chua's circuit which uses a coaxial cable in place of the LC tune circuit; there is a microwave frequency version of Chua's circuit which can be modeled by a piecewise-linear 1-D map with a time delay; there is a 1-dimensional array of coupled Chua's circuits where signals can propagate, or be interrupted abruptly, depending on the coupling resistance; there are cellular neural networks made of Chua's circuits which can display rather complex dynamical patterns.

There are papers which propose some novel applications of chaos, including pattern recognition, low-level visual sensing, and the generation of musical tones. There are many papers on synchronization and on controlling chaos in Chua's circuits.

In addition to the 56 papers listed in the Table of Contents, a computer diskette containing a user-friendly software package entitled **ABC**: **A**dventures in **B**ifurcation and **C**haos comes with the book. Developed by M. P. Kennedy and C. W. Wu, the **ABC** package contains a collection of user-friendly and menu-driven computer programs. It contains an extensive data base library of exact parameter values and initial conditions needed to reproduce many of the bifurcation and chaotic phenomena described in this book.

In addition to **ABC**, the diskette also contains a software package called **CHUA** (written in FORTRAN by M. Biey, S. Chialina, M. Hasler, and A. Premoli) which exploits the *piecewise-linear* nature of Chua's equations for a more accurate and efficient calculation of time series and Lyapunov exponents.

A milestone in the research on Chua's circuit has been achieved with the recent design of the *first* CMOS monolithic Chua's circuit which makes it possible to produce an entire array of Chua's circuits in a single chip smaller in size than a fingernail. This highly significant seminal work proves that a *chaos signal generator* can be mass produced as off-the-shelf electronic components, just like such standard IC components as diodes, transistors, op amps, etc.

In view of the great diversity of the topics covered and the unusually large number of authors coming from different parts of the world, it is inevitable that there will be some ambiguities in terminologies. Whenever time permits, we have asked the authors to revise their manuscripts in order to conform to the following definitions, which we hope will become *customary* in future publications on Chua's circuits:

Suggested Terminologies and Notations for Chua's Circuit:

1. *The 5-element circuit will continue to be called* **Chua's circuit**. *However, the original v–i characteristic of* **Chua's diode** *may assume* **any** *single-valued, (not necessarily symmetric or piecewise-linear) piecewise smooth function. The recently augmented Chua's circuit obtained by connecting one linear resistor in series with the inductor will be called the* **Chua's oscillator**, *except in the context of topological conjugacy when it may also be called* **a canonical Chua's**

circuit. *Since there exist several distinct circuits which are also topologically conjugate to Chua's circuit, their circuit diagrams must always be specified to avoid being confused with the canonical Chua's circuit as defined above.*

2. *The state equations associated with Chua's circuit will be called* **Chua's equations** *in both the physical and the dimensionless forms. The two dimensionless parameters will be denoted by* α *and* β*, respectively. The equations associated with the Chua's oscillator will be called the* **canonical Chua's equations**. *The three dimensionless parameters will be denoted by* α*,* β*, and* γ*, respectively.*
3. *The original 1-D map associated with Chua's circuit will be called* **Chua's 1-D map**.
4. *Two strange attractors observed in the Chua's circuit have been the spiral Chua's attractor and the double-scroll Chua's attractor. Many more have now been observed in the Chua's oscillator. All these will be called a* **family of Chua's attractors**.

This preface would not be complete without a brief history of Chua's circuit, which was originally conceived and designed by L. O. Chua in 1983. Following instructions given by Chua, T. Matsumoto wrote an *ad hoc* computer program, verified its chaotic nature, and named it Chua's circuit in his seminal publication in December 1984. The experimental confirmation of chaos from Chua's circuit was published in January 1985 by G. Q. Zhong and F. Ayrom. The brute-force computations involving hundreds of hours of computer time were carried out by Matsumoto and his team of dedicated students. The eigenspace analysis and the reduction to the real Jordan form were made by M. Komuro. Professor L. O. Chua informed me that he did not participate in the early phase of the research due to a sudden illness which occurred shortly after he had explained to Matsumoto on how the circuit works, and how to pick the appropriate parameters to attain a chaotic regime. Unlike most other chaotic systems, Chua's circuit was designed using a systematic nonlinear synthesis technique developed by Chua. For an insightful account of the design procedure, as well as the episode which led to Chua's circuit, the reader is referred to the following historical paper:

L. O. Chua, "The genesis of Chua's circuit", *Archiv für Elektronik und Übertragungstechnik* 46 (1992) 250–257.

Readers interested in the latest results on Chua's circuit are referred to *three* Special Issues of the *IEEE Transactions on Circuits and Systems* on "Chaos in Nonlinear Electronic Circuits", (October and November 1993).

Throughout the preparation of this book I have been inspired by the many conversations with Professor Leon O. Chua and in fact this book should be considered a product of his innate ability to project and share the creativity of his work with his fellow scientists. I am thankful to him for bringing the fascination of this subject to me as he is known to have done it for many of his students and collaborators.

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