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## PREFACE

In recent years there has been a strong increase of interest in strongly correlated electronic systems in the Condensed Matter community. The reason for this development is that certain phenomena like high- $T_c$  superconductivity cannot seem to be explained in the framework of weak interactions, that is mean-field theory and perturbation around it. There are mainly two different approaches in dealing with strong correlations: numerical studies and exact analytical solutions. Analytical solutions provide a complete and unambiguous picture of the dynamics of the models under consideration. The analytical approach is mainly limited to one spatial dimension, but there are indications that two-dimensional systems may share certain features with their one-dimensional analogs (see e.g. P. W. Anderson in Ref. 1; for a modified Hubbard model it was recently shown that the ground state structure is identical in one and two dimensions<sup>2</sup>). This volume is devoted to exact solutions of models of strongly correlated electrons in one spatial dimension by means of the Bethe *Ansatz*.

The first such exactly solved model is the non-relativistic continuum model of electrons with local interaction, solved by C. N. Yang in 1967<sup>3</sup> (see also Ref. 4). In the solution of the model the nested Bethe *Ansatz*, which is the basis for all exact solutions of electronic models in one dimension, was discovered.

The most important model of strongly correlated electrons, and the central topic of this volume, is the Hubbard model. The first chapter is devoted to its exact solution in one space and one time dimension, which started with E. H. Lieb's and F. Y. Wu's work of 1968. The chapter starts with reprints dealing with the construction of eigenstates of the Hubbard hamiltonian and the determination of the ground state and excitation spectrum at zero temperature. The next few topics are the study of magnetic properties in an external magnetic field, the thermodynamics of the model, and the asymptotic behaviour of correlation functions. The chapter closes with reprints on transport properties of the model.

The second chapter is devoted to the  $t$ - $J$  model at the supersymmetric point  $J = \pm 2t$ . The  $t$ - $J$  model first appeared as an effective model describing the strong coupling limit of the Hubbard model for  $H \ll t$ ). Later the model was reinvented to describe copper-oxide planes in high- $T_c$  superconductors. Nowadays the  $t$ - $J$  model is the most thoroughly studied model of strongly correlated electrons after the Hubbard model. The organization of Chapter II is similar to that of Chapter I.

The third chapter is about other solvable models of strongly correlated electrons. The first half of the chapter is devoted to various electronic models with local interaction — the nonrelativistic continuum model of electrons with local interac-

tion, the Luttinger model, and models which were constructed recently in relation to high- $T_c$  superconductivity. The second part of the chapter deals with models with long-range interactions, which have recently attracted much attention in relation to fractional statistics.

The reprinted papers are complemented by a list of some 230 references at the end of the volume. In this list reprinted papers are marked by a bullet ( $\bullet$ ). When referring to reprinted papers in the comments we first give the chapter number and then the number of the reprint within the chapter, e.g. [repr.III.A.2] would refer to the paper by M. Takahashi reprinted in part A of chapter III under the number 2.

The central topic of this volume is lattice models of strongly correlated electrons. We did not include reprints on models describing interactions of electrons with impurities (see the excellent reviews in Refs. 5, 6) or other excitations in this volume. We also did not include papers dealing with purely mathematical aspects of integrable models. There are many excellent books dealing with these issues<sup>7-16</sup> and we refer the interested reader to them. We also did not include any reviews on the Hubbard model.

Due to republication fees charged by certain journals we were not able to include a significant number of papers which we had originally planned to get reprinted. We are very sorry about this unfortunate development but are confident that the volume in its current form will still be helpful for researchers and students.

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