

and in the backward direction

$$\frac{d\sigma_+}{d\Omega} = 4|B|^2 \quad \frac{d\sigma_-}{d\Omega} = 2(|A|^2 + |B|^2) \quad (29)$$

c) Under parity the following transformations take place

$$\boldsymbol{\sigma} \rightarrow \boldsymbol{\sigma} ; \mathbf{p} \rightarrow -\mathbf{p} ; \mathbf{p}' \rightarrow -\mathbf{p}' \quad (30)$$

Parity invariance, together with invariance under a rotation by π on the scattering plane, implies the following relation among helicity amplitudes

$$f_{h_1, h_2; h'_1, h'_2} = f_{-h_1, -h_2; -h'_1, -h'_2} \quad (31)$$

Problem 4. Scattering asymmetry due to $\ell \cdot s$ interaction

The interaction of a spin-1/2 particle in a potential $V(r)$ has the form

$$V(r) + \frac{g}{2m^2} \frac{1}{r} \frac{dV}{dr} \boldsymbol{\ell} \cdot \mathbf{s} \quad (1)$$

The spin-orbit term has its origin in relativistic corrections. We consider a model in which the complete interaction is given by $U = V + V'$, where

$$V(r) = -V_0 \theta(r_0 - r) \quad V_0 > 0 \quad (2)$$

and V' is a spin-independent, imaginary interaction simulating inelastic processes

$$V' = -iV_1 \theta(r_0 - r) \quad (3)$$

- a) Calculate the scattering amplitude in the Born approximation.
- b) Calculate the left-right asymmetry in the scattering of transversely polarized particles, and the induced polarization on nonpolarized particles.

Solution

a) In the Born approximation we write

$$f(\theta) = -\frac{m}{2\pi} \langle \mathbf{p}' | U | \mathbf{p} \rangle = -\frac{m}{2\pi} \int e^{-i\mathbf{q}\mathbf{r}} U(r) d^3\mathbf{r} \quad (4)$$

where $\mathbf{q} = \mathbf{p}' - \mathbf{p}$ is the momentum transfer.

We calculate the spin-independent part making use of the relation

$$\int e^{-i\mathbf{q}\mathbf{r}} \theta(r_0 - r) d^3\mathbf{r} = \frac{4\pi r_0}{q^2} \left(\frac{\sin qr_0}{qr_0} - \cos qr_0 \right) \quad (5)$$

The spin-orbit part is calculated by means of

$$\begin{aligned} \langle \mathbf{p}' | \delta(r - r_0) \boldsymbol{\ell} \cdot \mathbf{s} | \mathbf{p} \rangle &= \int d^3\mathbf{r} e^{-i\mathbf{p}'\mathbf{r}} \delta(r - r_0) \frac{1}{i} (\mathbf{r} \wedge \nabla) \cdot \mathbf{s} e^{i\mathbf{p}\mathbf{r}} \\ &= \int d^3\mathbf{r} e^{-i\mathbf{q}\mathbf{r}} \delta(r - r_0) (\mathbf{r} \wedge \mathbf{p}) \cdot \mathbf{s} \\ &= i \frac{4\pi r_0^2}{q^2} \left(\frac{\sin qr_0}{qr_0} - \cos qr_0 \right) \boldsymbol{\nu} \cdot \mathbf{s} p^2 \sin \theta \end{aligned} \quad (6)$$

where

$$\boldsymbol{\nu} = \frac{\mathbf{p} \wedge \mathbf{p}'}{|\mathbf{p} \wedge \mathbf{p}'|} \quad \text{and} \quad |\mathbf{p} \wedge \mathbf{p}'| = p^2 \sin \theta \quad (7)$$

Thus, the complete amplitude reads

$$\begin{aligned} f(\theta) &= [2m(V_0 + iV_1) - i \frac{gV_0}{m} p^2 \sin \theta \boldsymbol{\nu} \cdot \mathbf{s}] \frac{r_0}{q^2} \left(\frac{\sin qr_0}{qr_0} - \cos qr_0 \right) \\ &= A + B \boldsymbol{\nu} \cdot \boldsymbol{\sigma} \end{aligned} \quad (8)$$

with

$$\begin{aligned} A &= \frac{2mr_0}{q^2} (V_0 + iV_1) \left(\frac{\sin qr_0}{qr_0} - \cos qr_0 \right) \\ B &= -i \frac{gr_0 V_0 p^2 \sin \theta}{2mq^2} \left(\frac{\sin qr_0}{qr_0} - \cos qr_0 \right) \end{aligned} \quad (9)$$

We have set $\mathbf{s} = \boldsymbol{\sigma}/2$, where $\boldsymbol{\sigma}$ are the Pauli matrices.

b) The left-right asymmetry, $\varepsilon(\theta)$, of transversely polarized particles is defined as follows

$$\varepsilon(\theta) = \frac{d\sigma(\theta) - d\sigma(-\theta)}{d\sigma(\theta) + d\sigma(-\theta)} \quad (10)$$

$\pm\theta$ is the scattering angle on the plane perpendicular to the initial polarization

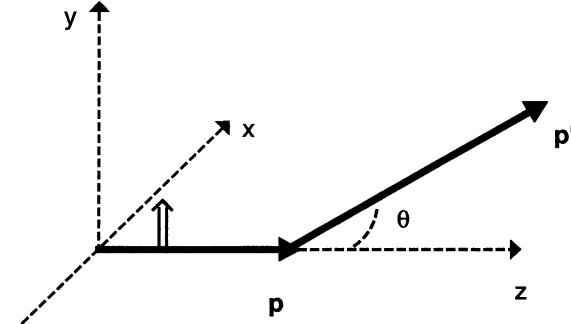


fig. 1

Denoting by P_{\perp} the transverse polarization of the beam, we have

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{1}{2} \text{Tr} \left\{ f(\theta) (1 + \mathbf{P}_{\perp} \cdot \boldsymbol{\sigma}) f^{\dagger}(\theta) \right\} \quad (11)$$

Substituting the expression (8) for $f(\theta)$ leads to

$$\varepsilon = \frac{2 \text{Re} A(\theta) B^*(\theta)}{|A|^2 + |B|^2} = - \frac{2 V_0 V_1 p^2 g \sin \theta}{4 m^2 (V_0^2 + V_1^2) + \frac{g^2 V_0^2 p^4}{4 m^2} \sin^2 \theta} \quad (12)$$

The polarization induced on nonpolarized particles and angle θ is obtained through

$$\mathbf{P} = \frac{\text{Tr} (\boldsymbol{\sigma} f(\theta) f^{\dagger}(\theta))}{\text{Tr} (f(\theta) f^{\dagger}(\theta))} = \frac{2 \text{Re} AB^*}{|A|^2 + |B|^2} \boldsymbol{\nu} = \varepsilon \boldsymbol{\nu} \quad (13)$$

We observe that this equals the asymmetry calculated above, as would be expected by time reversal invariance.

Problem 5. The "static" photoelectric effect

A metal can be represented by a potential well, filled with free electrons. Let n be the number of electrons per unit volume and V_0 the extraction potential. (Typically, $n \approx 10^{23}/\text{cm}^3$; $V_0 \approx 2$ Volts.)

- How deep is this potential well?
- Suppose now the metal is illuminated by light with intensity I and frequency ν ($h\nu > eV_0$). What is the energy distribution of the emitted photoelectrons?
- If this metal is the negative shield of a capacitor, a static photoelectric effect may take place: the presence of the electric field causes electrons to cross the barrier which binds them to the metal.

Calculate the current density of these electrons.

Solution

- The number of electrons in the metal is

$$N = \frac{2V}{h^3} \int_0^{P_F} d^3 \mathbf{p} \quad (1)$$

The factor of 2 is due to spin degeneracy and V is the volume of the metal. The corresponding density is

$$n = \frac{8\pi}{3} \frac{1}{h^3} P_F^3 \quad (2)$$

Knowing the density, we can deduce the value of the Fermi momentum P_F .