

# Foreword

This book could be called *Mathematical ecology beyond the Lotka-Volterra model* and we might add: *far beyond*. In theoretical biology and applied mathematics, the logistic and Lotka-Volterra models have long been considered as seminal examples of modeling and dynamics. However, it was understood only recently how different forms of regulatory mechanisms, like birth and death, competition, consumption and the like, result in changes of the stability and dynamics of ecological systems. The present book brings this understanding to the attention of a broad biological and nonlinear dynamics audience. It does so with a deep and unique insight into the mathematical richness of basic ecological models and how this richness emerges as the number of competing mechanisms or factors (reflected in the number of parameters, not state variables) increases.

The main topics of the book are:

- the dynamics of elementary ecological communities, consisting of two or three trophic levels;
- the stabilizing and destabilizing role of various regulatory mechanisms determining the outcome of ecological interaction within the community;
- “dangerous boundaries” for the stability of the ecosystem and criteria for approaching them.
- mechanisms of spatial inhomogeneity and their relationship to non-equilibrium dynamics of ecosystems.

The strength of this book is that it systematically builds a sequence of well-motivated ecological models of increasing difficulty and classifies them with methods from bifurcation theory. To this end, the author emphasizes the use of higher order degeneracies. This makes this book quite unique and interesting not only for a biological audience, but also for the applied dynamical systems community. In fact, this text can be used as a guided tour to bifurcation theory from the applied point of view. The interested reader will find a wealth of intriguing examples of how known bifurcations occur in (biological) applications.

There is a clear structure throughout, and we feel that it will be of help to the reader to sketch it here. All models, especially in the analysis of two-species ecosystems, are put into a matrix-like structure reflecting the interaction between different stabilizing and destabilizing factors. Each model is then reduced by scaling to a convenient form and is analyzed by means of bifurcation theory. A complete description of the bifurcation diagram is given and illustrated in the figures. The phase portraits and bifurcations are then explained in detail with the emphasis on changes in phase or parameter space that lead to qualitatively different behavior. In a last step, rewarding especially for those who are primarily interested in the biological implications, this bifurcation analysis is interpreted from the ecological point of view. Each chapter has an appendix (not included in the Russian edition) containing numerically computed phase portraits together with the equations and parameter values. When compared with the figures in the text, they give an impression of the physical appearance of the systems. The reader is encouraged to investigate the models with any simulation program.

The bulk of the material in this book is based on original research that the author conducted with his collaborators in the 1970s and 1980s at the Institute of Mathematical Problems in Biology in Pushchino, Russia. It was originally published in Russian in 1985. The author worked on an English translation of the original in its revised and extended form, but due to his tragic death in 1994 he could not complete this project. We have assumed the role of translation editors in an effort to finish the project. Our editing consisted essentially in making the text more accessible by using a language of modern bifurcation theory that we consider fairly standard. We tried to keep the spirit of the original as much as possible and included the original preface together with a biographical introduction provided by the author's family.

We thank Elena P. Kryukova (Bazykina), Yegor A. Bazykin and Dmitry A. Bazykin for their continuous support and, in particular, for the preparation of the appendices. For their encouragement and advice on our editing, we thank Faina S. Berezovskaya, John Guckenheimer, Alexey S. Kondrashov and Emmanuil E. Shnol. Finally, we thank the editorial staff at World Scientific for their good cooperation.

*Ithaca/Amsterdam*  
*June 1997*

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