the energy for producing the Rindler particles, which an accelerated micro-detector would register, might come from the accelerator itself, since as it accelerates the micro-detector, "the work done by [it] ... supplies the missing energy that feeds into the [quantum] field via the quanta emitted from the detector." (Birrell and Davies, 1982, p. 55.) But in that case the whole Rindler particle production phenomenon is due to the fact that an open system, capable of receiving energy from the outside (i.e., from the accelerator), was treated as a closed system – whereas the physically *correct* treatment should have incorporated from the outset all the quantum fields created by the accelerator during the acceleration process. In that case the same argument cannot be applied to a similar micro-detector in *free* fall[4], despite all the manifest formal analogies between Rindler particle production and the particle creation *ex nihilo*, which are very much stressed in some of the literature on the subject. Indeed, if the strong equivalence principle of general relativity is correct, then *"observers" in free fall are truly inertial* – and as such they are not to be viewed as being accelerated by an external gravitational field, as is the case in Newtonian mechanics.

On the other hand, if it is assumed that a freely falling observer in curved spacetime is, contrary to Einstein's point of view, an accelerated observer, the question arises: in relation to what is he accelerated? The only *mathematical* answer to that question that can be found in the literature is that, in the case of curved spacetimes that are *asymptotically* flat, an observer in free fall is accelerated in relation to an observer in the *fictional* flat spacetime that asymptotically merges with the considered curved spacetime. However, since no acceptable cosmological models of our universe as a whole are asymptotically flat, even such an *ad hoc* "solution" does not provide a truly satisfactory answer.

These fundamental difficulties of the conventional framework for quantum fields in curved spacetime extend also to its formulation of the vacuum state. Indeed, it is again acknowledged in standard literature on the subject that "as far as Minkowski space is concerned, the [Fock] vacuum is a strong candidate for the 'correct' or 'physical' vacuum – the experiences of the accelerated observers being 'distorted' by the effects of their non-uniform motion. The trouble is that when gravitational fields are present, inertial observers become free-falling observers, and in general no two free-falling detectors will agree on a choice of vacuum." (Birrell and Davies, 1982, p. 55.)

We can infer from all this that the above mentioned "lack of agreement" between various free-falling detectors is due to the fact that in CGR all inertial observers are *local* observers, whereas the only "vacuum" considered in conventional quantum fields in curved spacetime is a *global* vacuum. Hence, as part of the solution to all of the above fundamental inconsistencies, we shall offer from Chapter 5 onwards a quantum geometry framework for QFT based on fibre bundles whose fibres are Fock spaces, so that *all* Fock vacuum states will be local rather than global.

1.6. From Canonical Quantum Gravity to Superstrings

The opinion that Einstein's theory of gravity has to be quantized is very widely held, but there are exceptions: Møller (1952) and Rosenfeld (1957) have argued that the gravitational

[4] Cf. Sec. 2.6 for a discussion of the geodesic postulate and of the strong equivalence principle governing all free-fall conditions in CGR.

field should not be quantized at all, and more recently arguments to the same effect have been presented in a monograph which ends with the following conclusion:

"The limitations on measurement of gravitational fields are, due to the geometrical character of gravity, necessarily limitations on the size of spacetime regions over which the Bohr-Rosenfeld quantum measurement procedure is, in principle, feasible. One way of expressing this fact is to say that supplementary to the well-known nonlocalizability of classical, i.e., non-quantized gravitational fields (making the $L_0 \to 0$ limit for lengths L_0 [of test bodies] physically senseless), there exists a finite limit ℓ_P [i.e., the Planck length] on the localizability of quantized gravitational fields. This is owed to the fact that localizability is inconsistent with Heisenberg's uncertainty principle taken in conjunction with the basic tenets of general relativity." (von Borzeszkowski and Treder, 1988, p. 100.)

Whereas an inconsistency between Heisenberg's uncertainty principle and such basic tenets of general relativity as the general covariance and strong equivalence principles is indeed in evidence if classical Lorentzian geometries are adopted for the description of spacetime, we shall demonstrate in Chapter 8 that this is no longer the case if appropriate *quantum* geometries are adopted instead. Hence, the simple and direct argument in favor of quantizing gravity that was given by Dirac remains in effect: "There is no experimental evidence for the quantization of the gravitational field, but we believe quantization should apply to all fields of physics. They all interact with one another, and it is difficult to see how some could be quantized and others not." (Dirac, 1968, p. 539.)

Dirac's (1950, 1958, 1959) pioneering work on Hamiltonian dynamics with constraints had prepared the ground for the canonical quantization of gravity, but Dirac himself did not further pursue this quantization since he eventually reached the following conclusion: "There does not exist any general method for handling [in quantum gravity] quadratic quantities in the δ-function, free from inconsistencies. ... The problem of the quantization of the gravitational field is thus left in a rather uncertain state. If one accepts Schwinger's plausible [but nonrigorous] methods, the problem is solved. But one cannot be happy with such methods without having a reliable procedure for handling quadratic expressions in the δ-function." (Dirac, 1968, p. 543.)

The origin of some of these and other inconsistencies can be traced to the presence of constraints in the canonical formulation of gravity. Thus, in trying to adapt the *non*-relativistic canonical quantization scheme to the general relativistic regime, it has been postulated that "in the quantum version of a general-relativistic theory only observables should play a role." On the other hand, in CGR "only first-class variables are observable" (Bergmann and Komar, 1980, p. 243) – where, in accordance with Dirac's (1959, 1964) classification, a first-class canonical variable is defined as one whose Poisson brackets with the Hamiltonian constraints vanish identically. However, as pointed out by Bergmann and Komar (1980) in Sec. 5 of their review paper on the subject,[5] such a definition removes the status of "observable" from the most basic of Einstein's actually observable quantities in CGR, namely from the metric tensor. To rectify this glaring inconsistency with orthodox

[5] Cf. also Bergmann's articles in (Bergmann and de Sabbata, 1986) and (de Sabbata and Melnikov, 1988). In Sec. 2.2 we shall discuss a concept of "metrization" of bundles of consisting of frames of reference, which can be described in operational terms that are physically tantamount to measurements of classical metrics.

theory, the artificial concept of "quasiobservable" has to be introduced, so that the nonrelativistic position operators can be then reinstated at least to the partial status of "quasiobservables," and so that the conjecture can be then advanced that in canonical gravity the twelve canonical variables are "quasiobservable."

In light of the renewed interest in canonical quantum gravity due to the discovery of new variables (Ashtekar, 1991), the question has to be therefore asked as to whether some new kind of "observables" in the sense of "smooth functions on the phase space of the theory [which, if physical] have vanishing Poisson brackets with the constraints" (Rovelli, 1991b, p. 301) might exist in CGR, despite the fact that in the case of "the vacuum Einstein equations in the spatially compact case, *not a single physical observable is known explicitly as a function of phase space variables*." (Smolin, 1991, p. 447.) This question is especially pertinent in light of the fact that the italicized part of this statement represents a reiteration of the observation that "one obvious difficulty with this [kind of] approach is that thus far no one has been able to discover a single classical observable" (Bergmann and Komar, 1980, p. 246), which would be represented by a first class variable on the phase space of a CGR theory. Since the latter statement was actually made more than a decade prior to the previous one, this lack of explicit examples is obviously not due to the lack of serious attempts at trying to find such "physical observables." Rather, it reflects some deeper features of general relativistic theories, intimately related to their underlying physical nature.

Of course, the motivation behind the renewed search for some "physical observables" invariant under the diffeomorphism group *Diff* **M** of a Lorentzian spacetime manifold **M** does not reside in the interpretation of CGR, but in their conceived usefulness to the new approach to the canonical quantization procedure based on "loop representations." These types of approaches are based on purported "quantum observables" represented by "diffeomorphism-invariant operators", for which, however, "the problem is that we do not know how to make a correspondence between any of them and the classical diffeomorphism-invariant observables." (Smolin, 1991, p. 468.) On the other hand, the *true* observables of CGR, which have served Einstein (1916) in the prediction of *all observable* effects in CGR, are *not* the elements of the Lorentzian spacetime manifold, *nor* its differential-manifold structure, and *generically not* even the holonomic metric components $g_{\mu\nu}$. As discussed by M. Friedman (1983), these are all indispensable *mathematical* objects, reflecting the "style" in which CGR is formulated, but none of them have any direct physical significance. Rather, the basic observables in CGR are the *non*holonomic metric components g_{ij} with respect to classical *local* frames, the components of various tensors, such as the stress-energy, angular momentum, matter and nongravitational radiation fields with respect to the same frames, and the *relative* positions of the test particles which are in the immediate neighborhoods of the locations of such frames – with those in free-fall playing an especially important role as providers of the best-known observable predictions in CGR that are given a prominent place in all textbooks on general relativity.

An entirely different type of approach to the quantization of gravity (DeWitt, 1967) has become known under the name of covariant quantum gravity. B. S. DeWitt, who has made fundamental contributions to both the canonical and the "covariant" approach, has remarked in one of his key 1967 papers on the subject that "... no rigorous mathematical link has thus far been established between the canonical and covariant theories. In the case of infinite worlds it is believed that the two theories are merely two versions of the same theory, expressed in different languages, but no one knows for sure." In essence, DeWitt's 1967 assessment has remained valid to the present day.

The "covariant" approach to quantum gravity assumes an already given Lorentzian manifold, so that the components of the quantum metric field can be then viewed as equal to a sum whose first term is a "classical" Lorentzian metric on that manifold (called in that context the "background metric"); whereas the second term is deemed to represent a "quantum correction," and is formally viewed as operator-valued function on the given manifold **M**. Since such a decomposition is obviously not invariant under the diffeomorphism group *Diff* **M**, covariant quantum gravity is actually not general relativistically covariant in the modern sense (discussed in Sec. 2.2). Nevertheless, in the 1970s this approach, with a choice of background metric equal to the Minkowski metric, enjoyed greater popularity than the canonical approach, since general coordinate invariance could be then *formally* embedded into the framework via a non-Abelian gauge group, so that the *formal* perturbative techniques based on the Feynman rules developed for Yang-Mills theories could be applied to it (Duff, 1975, 1981; van Nieuwenhuizen, 1977).

Many questions could be raised regarding the physical justifiability as well as the mathematical validity of this approach. However, from the conventional point of view its fatal fault lies in the nonrenormalizability of the terms in its formal perturbation expansion, so that the following conclusion was ultimately reached: "The failure to combine the particle physics version of quantum theory and general relativity poses a fundamental problem, since gravitation undeniably exists as a force in nature. Either our quantum theory must be modified, or other gravitational models should be considered, or we must leave gravitation unquantized, which might be inconsistent according to a Bohr argument." (van Nieuwenhuizen, 1977, p. 24).

The later work of Goroff and Sagnotti (1986) on two-loop formal perturbative expansions in pure quantum gravity confirmed the impossibility of arriving at even formally renormalizable expressions. In the meantime, hopes were entertained that supergravity (Wess and Bagger, 1983; West, 1986) might be able to supply models constructed in the same vein, and which would be renormalizable in the conventional sense. However, those expectations were not fulfilled (cf. Ashtekar, 1991a, p. 7).

For these reasons, during the last decade most of the hopes of researchers in quantum gravity were pinned on superstring theory. This was due to the fact that, at the level of formal perturbation theory, the vertices of diagrams in string theory contain exponential nonlocal factors that cause loops to converge in the Euclidean regime. However, it turned out that superstring perturbation theory encounters a breakdown of (its originally presumed) uniqueness after the compactification to four spacetime dimensions is carried out (Narain, 1986). Moreover, it was ultimately proven that its perturbation series not only is *not* convergent, but it is not even Borel summable (Gross and Perival, 1988).

In view of these and other problems, by the end of the 1980s, the following essential points were made in a review of quantum gravity, written by an advocate of string theory: "It is true that we are not (yet) able to address the physically more interesting questions in quantum gravity. But this is mainly due to lack of technique (and probably also lack of some deep understanding of the fundamental physical principles)." (Alvarez, 1989, p. 602.)

On the other hand, the route for eventually arriving at such a "deep understanding of fundamental physical principles" cannot be supplied by string theory itself, since there is a huge methodological gap between the manner in which string theory originally emerged from the Veneziano model in the late 1960s, and was subsequently developed in the course of the 1970s and 1980s, and the manner in which classical general relativistic gravity theory was created and developed by Einstein. The existence of this gap is actually pointed out in one of the main textbooks on string theory:

"For a theory that makes the claim of providing a unifying framework for all physical laws, it is the supreme irony that [superstring] theory itself appears so disunited, ... [with] the fundamental physical and geometric principles that lie at [its] foundation... still unknown. ... By contrast, when Einstein first discovered general relativity, he started with physical principles, such as the equivalence principle, and formulated them in the language of general covariance." (Kaku, 1988, pp. viii, 5-6.)

This monograph is therefore devoted to arriving at an understanding, at a physical as well as mathematical level, of those fundamental principles in general relativity and quantum theory that can lead to a purely geometric framework for their unification. This framework will be based on a systematic but very careful extrapolation to the quantum regime of the principles enunciated by Einstein in developing classical general relativity, and of an extrapolation to the general relativistic regime, carried out in the same spirit, of the principles enunciated by Bohr, Born, Dirac, Heisenberg, and other well-known founders of quantum mechanics as they developed fundamental theoretical results that have withstood the test of time. For, as will be seen in the subsequent chapters of this monograph, the mathematical tools that were not available to them when they formulated those principles have been made available by developments in the theories of gauge fields, gauge groups, coherent states, fibre and superfibre bundles, supermanifolds, etc., carried out during the past couple of decades.