PREFACE

It is now well accepted that chaos is an ubiquitous and robust nonlinear phenomenon frequently encountered in nature. During the past two decades or so the concept of chaos has permeated almost all branches of science and engineering. The field is growing into a stage where the initial surprises associated with the phenomenon are waning and new understandings are appearing, while actual controlling and harnessing of it are being contemplated. In these developments, the study of nonlinear oscillators has played a very important role in understanding the chaotic phenomenon. Many ubiquitous systems such as the Duffing oscillator, damped and driven pendulum, driven van der Pol oscillator, and so on have been treated as paradigms in chaos research. The study of such systems is based mostly on approximate analytic approaches and detailed numerical investigations. From another point of view, nonlinear electronic circuits complement these studies through analog simulations. Besides, many new nonlinear electronic circuits have been constructed, which are dynamical systems of interest on their own accord. Chua's diode and related circuits are foremost examples of nonlinear electronic circuits which act as veritable black boxes to study nonlinear phenomena. Thus the twin approaches of numerical analysis and circuit theoretic studies can complement each other in the investigation of bifurcation and chaos phenomena in nonlinear dynamical systems.

In recent times, one has witnessed considerable activity in the controlling of chaotic motions to desired regular orbits, through predetermined small perturbations. Various algorithms have been proposed and implemented successfully to avoid the harmful effects of chaos, when required, and bring back the system to desired regular states by minimal changes. But more surprisingly chaos can also be harnessed in a purposeful way leading to exciting technological applications. Against common beliefs, identical chaotic systems can be synchronized provided an appropriate coupling is introduced between them. This in turn leads to the possibility of spread-spectrum secure communications of both analog and digital signals.

The aim of this book is essentially to analyse the bifurcation and chaos phenomena in typical nonlinear oscillators, especially of damped and driven types, from both dynamical and circuit theoretical points of view, and then to introduce the concept of controlling and synchronization in them. Though many important books on chaotic dynamics have appeared in the recent literature stressing different aspects, the authors believe that the approach taken in this book and the topics covered deal with many aspects not readily discussed in other books on chaos.

Specifically, after giving a brief introduction to the topic of nonlinear dynamics in Chapter 1, we introduce the elementary notions on the dynamics of linear and nonlinear oscillators in Chapter 2. In Chapter 3, a brief introduction to linear and nonlinear circuit theory is provided and the relation to dynamical systems is explained. Bifurcation and chaos phenomena with specific reference to the Duffing oscillator are discussed in Chapters 4 and 5. The different types of attractors, bifurcations, and routes to chaos are discussed in detail for the double-well, single-well and double-hump Duffing oscillators by both numerical analysis and analog circuit simulation in Chapter 4. Complementing these studies, an analytic investigation of the Duffing oscillator is carried out in Chapter 5 through approximate (perturbation and linear stability) analyses, Melnikov criterion and analytic structure (Painlevé singularity structure) studies.

Chapter 6 deals with the bifurcation and chaos aspects of the Bonhoeffer–van der Pol (BVP) and Duffing–van der Pol oscillators, involving numerical, analytical and analog simulation studies. Chapter 7 is devoted to a study of bifurcation and chaos phenomena in nonlinear electronic circuits involving piecewise-linear Chua's diode by both experimental and numerical analyses. We consider the behaviour of Chua's oscillator, the autonomous Chua's circuit, the driven Chua's circuit, the simplest dissipative nonautonomous circuit, and the autonomous Duffing–van der Pol oscillator here.

The final part of the book, consisting of Chapters 8 and 9, deals with some very recent developments in chaotic dynamics, namely controlling of chaos and synchronization of chaotic systems. In Chapter 8 we give a brief account of the various algorithms suggested for controlling of chaos and apply these algorithms to the BVP oscillator as a test case. Some of the methods are also applied to the other oscillators mentioned above. Finally, in Chapter 9 we introduce the Pecora and Carroll method of chaos synchronization with and without cascading, as well as the alternative method of one-way coupling of identical chaotic systems. We then illustrate the possibility of transmitting analog and digital signals using synchronized chaotic signals as carriers in a secure way and apply it to the various oscillators discussed.

The book also contains three appendices on (i) perturbation methods, (ii) van der Pol oscillator, and (iii) some other standard oscillators. Also, a glossary of specialized terms is included.

In the absence of exact analytical methods, numerical studies alone cannot provide a complete picture of the dynamics in the parameter space. For most of the oscillators considered in this book, such phase diagrams given in the text cover only a portion of the parameter space. More extensive analysis is required to cover the entire space of the parameters. However, we do hope that the present book may motivate further work along this direction.

In our endeavour to write this book, we have received whole-hearted support from the members of the Nonlinear Dynamics Group at Bharathidasan University. We have freely used their research results in our discussions in the book. In addition, Dr. M. Daniel and Dr. S. Rajasekar helped us by providing critical comments on the manuscript. We thank them and other members of the group for their cooperation.

In the main body of the book, we have also used materials and figures from many published articles by different authors, which are referred to at appropriate places in the text. We thank the American Institute of Physics and Institute of Physics, U.K., for granting permission to reproduce some of the figures appeared in their journals. For a book of this nature, it is impossible to refer to all the related published literature. Though we have tried to give the relevant references which are familiar to us, we apologize to those authors whose work we did not mention due to either our ignorance or unfamiliarity.

Finally, we wish to record our acknowledgement of the continued support received from Bharathidasan University and the Department of Science and Technology, Government of India, for many years for our various activities in nonlinear dynamics. This support is the main inspiration for us to undertake the endeavour of writing this book.

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