

Preface

Main way of the development of the theory of the dynamical systems in the twentieth century is the investigation of attractors. In the first three decades of the present century the attention of the researchers was concentrated on the attractors consisting of the single stationary point. As a result, the classical stability theory was created. In the next three decades of the century the exploration of attractors consisting of stationary sets and limit cycles led to the creation of classical theory of oscillations. In the last third of our century the homoclinic and heteroclinic orbits as well as the strange attractors have been intensively investigated.

On the other hand in the fifties and the sixties of our century the frequency-domain methods of investigation of nonlinear systems were set up. At first they were applied and developed only within the frames of absolute stability theory for investigation of global stability of the stationary point. The development of frequency-domain methods was essentially stimulated by the property of invariance of the transfer function and frequency response with respect to linear transformations of the phase space. It should be emphasized in this connection that the form of description of dynamical system by means of transfer function, without participation of coordinates, is usual to engineers and is widely applied in engineering practice.

In the seventies and the eighties it became clear that the tool of frequency-domain methods can be applied successfully for the investigation of stability of stationary sets, for solution of problems of existence of cycles and homoclinical orbits as well as for the estimation of dimension of attractors. And this book is devoted to the consecutive exposition of this point of view.

The fundamentals of frequency-domain methods and of absolute stability theory are given in the two chapters of the first part. We tried to make our exposition as simple as possible here in order that every postgraduate familiar with the standard university courses of linear algebra, mathematical analysis and differential equations could master the fundamentals of frequency-domain methods. We tried to expound frequency-domain methods in spirit of excellent books by M. A. Aizerman and F. R. Gantmakher and by S. Levshetz. We took into account of course new results, contemporary trends and our own educational experience.

There exist two different methods in the generating of stability results in frequency-domain form. They are the application of Yakubovich–Kalman lemma about solvability of matrix inequalities and the constructing of Popov functionals.

The Yakubovich–Kalman theorem can be regarded as a generalization for the

nonlinear case of a famous theorem about the existence of Lyapunov function of the type of the quadratic form for a linear system with constant coefficients. That is why the Yakubovich–Kalman theorem is often used in the framework of direct Lyapunov method.

The Popov functional is the scalar product in the L^2 -space of two functions depending on solutions of the dynamical system. The elements of L^2 -space are transferred by means of the unitary operator (the Fourier transform) into more convenient for investigation space of images \tilde{L}^2 where the frequency response appears in a natural way. With the help of the latter the estimates of the scalar product in \tilde{L}^2 are brought about. Since the transform is unitary the same estimates are preserved in the space L^2 . The estimates give the opportunity to make various conclusions about stability and other qualitative properties of the system.

It is interesting to note that in spite of the difference of the tool of the investigation the both methods often give the same results. On the other hand for certain problems the success can be achieved only by one of them. Thus, the frequency-domain conditions of the existence of cycles and homoclinic orbits have been obtained by now only by means of Yakubovich–Kalman theorem, and the stability conditions of the synchronization system with delay have been established with the help of Popov functionals.

Proceeding from this, each of the first three parts of the book, which are devoted to stability of the unique equilibrium, to stability of stationary sets and to the problem of existence of cycles, contains two chapters. One of them demonstrates the application of Yakubovich–Kalman theorem and the other is based on the Popov method.

In the second part the problem of stability of stationary sets of a class of smooth dynamical systems is considered. By such class of dynamical systems many important classes of electric and electronic systems, such as Chua's circuits and systems of phase synchronization (phase-locked loops) are described. The latter have the cylindrical phase space. For these systems, in the book a special method of constructing of Lyapunov functions and Popov functionals is presented, the latter containing solutions of certain comparison systems. We shall note that continual stationary sets appear also in discontinuous relay systems and the systems with solid (Coulomb) friction. Frequency-domain investigation of these systems is expounded in the monograph [Gelig *et al.* 1978] where the theory of differential inclusions is systematically set forth. The limited capacity of this book made us confine ourselves only to smooth dynamical systems.

In the third part of this book the problem of the existence of cycles and of the estimation of their frequency is considered. The powerful tool of investigation of cycles is the Poincaré mapping. If one succeeds to prove that the Poincaré mapping transfers the transversal cross-section, on which it is defined, into itself, then by a certain theorem about a fixed point one can deduce the existence of a cycle. That is why the problem of existence of a cycle often can be reduced to the estimation of the Poincaré mapping. In the third part of the book various frequency-domain estimates of the Poincaré mapping are presented. For the systems with the cylindrical phase

space two kinds of the cycles are possible: the cycles of the first kind remain closed in the covering space, the cycles of the second kind lose the closure property there. Both for the cycles of the first kind and for the cycles of the second kind frequency-domain criteria of existence are obtained.

Analogous tool is used in the forth part in order to establish the frequency-domain criterion of existence of homoclinic orbits. For the systems with the cylindrical phase space. Such orbits appear when one transfers in the parameters space of the system from the region of gradient-like behavior to the region of existence of circular solutions and cycles of the second kind. We present in this book a frequency-domain criterion of the existence of a homoclinic orbit which can be regarded as a generalization of well-known Tricomi theorem about the existence of a separatrix loop.

In the fifth part the short review of basic notions of the dimension theory is brought about. Principal attention is paid here to the Hausdorff dimension which is one of the basic characteristics of strange attractors. We present analytical methods of obtaining of upper estimates of Hausdorff measure and dimension of attractors. Then frequency-domain estimates of Hausdorff dimension of attractors are demonstrated. These results are applied to the well-known Lorenz system.