Contents

Dedication vii
Contributors xi

Chapter 1: Stability of In-Phase and Out-of-Phase Modes for a Pair of Linearly Coupled van der Pol Oscillators
Duane W. Storti and Per G. Reinhall

1. Introduction 1
2. Derivation of Critical Variational Equation 5
3. Initial Inspection: Parameter-Independent Stability Transitions ... 6
4. Power Series Expansion for the van der Pol Limit Cycle 6
5. Power Series Expansion for the Periodic Variational Solution 8
   5.1 Displacement Coupling 8
   5.2 Velocity Coupling 10
6. Out-of-Phase Mode Analysis 11
   6.1 Existence of Out-of-Phase Oscillation 11
   6.2 Out-of-Phase Mode Stability 11
7. Results and Comparison with Numerical Simulation 13
8. Conclusion 20
9. References 21

Chapter 2: Perturbation Methods for Strongly Nonlinear Oscillators Using Lie Transforms
Vincent T. Coppola

1. Introduction 24
   1.1 Asymptotic Approximations 25
   1.2 Outline 26
2. The Lie Transform Method 26
   2.1 Application to Perturbation Methods 29
      2.1.1 Hamiltonian Systems 30
      2.1.2 Method of Averaging 31
   2.2 Implementation using Symbolic Computation 33

3. Averaging of the Duffing Oscillator 34

3.1 The Unperturbed System 35
3.2 Variation of Parameters 37
3.3 The Averaged System 39
3.4 Approximation of Limit Cycles 40
3.5 Comparison with Trigonometric Averaging 43

4. Synchronization of Hamiltonian Systems 46

4.1 The Polynomial Formulation 48
4.1.1 Example 49
4.2 The Action-Angle Formulation 52
4.2.1 The Cubic Oscillator 52

5. Concluding Remarks 54
6. Acknowledgements 57
7. References 57

Chapter 3: Momentum Transfer in Torque-Free Gyrostats
Christopher D. Hall

1. Introduction 60

2. Review of Related Publications 61

3. Equations of Motion 63

3.1 Derivation of Noncanonical Hamiltonian Equations 63
3.2 The Completely Integrable Case: $\epsilon = 0$ 68
3.3 Momentum Transfer Trajectories 71

4. Averaging for Small Constant Spinup Torques 74

5. Momentum Transfer Phenomena 77

5.1 Resonance Capture 77
5.2 Stationary-Platform Maneuvers 82

6. Conclusions 84

7. Acknowledgements 85

8. References 85

9. Appendix: Example Gyrostat Parameters 87

Chapter 4: The (Almost) Complete Dynamics of the FitzHugh Nagumo Equations
Dieter Armbruster

1. Introduction 89
2. The Normal Form 91
3. Reduction to Normal Form 92
4. Numerical Results 98
5. A Structurally Stable Neural Model 98
6. Conclusion 99
7. Acknowledgements 101
8. References 102

Chapter 5: Excitable Oscillators as Models for Central Pattern Generators
David Taylor, Philip Holmes and Avis H. Cohen

1. Introduction 103
2. A Two Oscillator Model 106
   - 2.1 $\alpha_j = 0$: Uncoupled Oscillators 106
   - 2.2 $\alpha_1 = \alpha_2 = \alpha$, small 107
   - 2.3 α large 108
   - 2.4 $\omega_1 = \omega_2 = \omega$ 109
   - 2.5 Simulations: Rotation Number vs. γ 109
3. Three and More Oscillators 111
   - 3.1 Chains with Varying Excitability 111
   - 3.2 Chains with Equal Excitability 113
   - 3.3 Towards a Model for the Observed Behavior 114
4. Discussion 115
5. Acknowledgements 117
6. References 117

Chapter 6: Solitons, Chaos and Modal Interactions in Periodic Structures
M.A. Davies and F.C. Moon

1. Introduction 119
2. Experiment 122
3. Numerical Model 123
4. Forced Vibrations and Model Interactions 128
   4.1 Numerical Experiment - Modal Trading 128
   4.2 Forced Vibrations of the Experimental Structure 130
5. Impact Response 135
   5.1 Comparison of Experiment and Model 135
   5.2 Calculation of Nonlinear Wave Speeds 139
6. Conclusions 140
7. Acknowledgements 142
8. References 142

Chapter 7: Two Applications of Nonlinear Normal Modes in Vibration Analysis
Melvin E. King, Johannes A.S. Aubrecht and Alexander F. Vakakis

1. Introduction 144
2. Nonlinear Standing Waves in a Layered System of Infinite Extent 144
   2.1 The Nonlinear Periodic System 145
   2.2 Standing Wave Solutions 146
3. Localized Periodic Oscillations in a System of Coupled Nonlinear Beams 156
   3.1 Experimental Fixture and Results 157
   3.2 Theoretical Model 162
4. Acknowledgements 166
5. References 166

Chapter 8: Friction as a Nonlinearity in Dynamics: A Historical Review
Brian Fenny and Ardéshir Guran

1. Introduction 168
2. Ancient Exploits of Frictional Awareness 169
3. Friction and the Theory of Dynamics 183
4. Friction in Nonlinear Dynamics 185
   - 4.1 Existence and Uniqueness of Solutions 185
   - 4.2 Friction-Induced Vibration 186
   - 4.3 Friction in Natural Phenomena 189
   - 4.4 Friction Damping 190
   - 4.5 Chaos and Nonsmooth Behavior 191
5. Conclusion 195
6. Acknowledgements 195
7. References 195

Chapter 9: A Quasiperiodic Mathieu Equation
Richard Rand, Randolph Zounes and Rachel Hastings

1. Introduction 203
2. Numerical Integration 204
3. Lyapunov Exponents 204
4. Regular Perturbations 204
5. Harmonic Balance 209
6. Singular Perturbations 212
7. Conclusions 216
8. References 220
9. Appendix 221

Author Index 222

Subject Index 227