

## CHAPTER 1

# A COMPUTATIONAL MARKET MODEL BASED ON INDIVIDUAL ACTION

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## 1. Introduction

There has been much recent interest in studying the implications of economic behavior by simulating markets. For example, the group at the University of Arizona used the PLATO computer network to conduct laboratory experiments with double auctions using human participants.<sup>19</sup> More recent results from a double auction tournament with computer trading programs is reported by Rust *et al.*<sup>14</sup> Smith<sup>16</sup> gives an overview of the literature on experimental economics. Work at Xerox PARC uses market-based ideas to allocate computer resources (see Huberman and Hogg,<sup>10</sup> Waldspuriger *et al.*,<sup>17</sup> for example). The term “computational ecology” has been used to describe this emerging field. Finally, recent work at Cal Tech by Ledyard *et al.*<sup>12</sup> and Porter and Rangel<sup>13</sup> uses computerized exchange mechanisms to allocate resources in a space exploration project.

Experimental markets with human subjects can lead to valuable insights into the mechanisms of competitive price determination, but it has some disadvantages. First, it is relatively expensive compared to computer simulations, and that limits the scope of possible scenarios that can be explored. Second, since human behavior is so complex, it is difficult to isolate the effect of one particular aspect of it. Of course we ultimately want to understand the economic implications of just such complicated behavior, but developing more controlled experimental techniques should yield additional insight.

The work described in this chapter is an attempt to provide a resource that lies between strictly analytical models and experimentation with human subjects. The approach provides a framework within which we can test theoretical results, and within which we can also observe the effects of changing individual aspects of hypothetical agent behavior. We will build a model of a micro-economy that is as simple as possible, while at the same time reflecting certain commonly accepted norms of behavior. For example, agents will have a need to acquire an essential commodity, which we call *food*, either by producing it themselves, or by buying it with a second commodity (*gold*) which they can produce. Our behavior-specifying algorithm is constructed so that an agent will bid higher if his inventory of food is low, and be willing to sell at a lower price if his inventory is high. This is not to say that such behavior always occurs in the real world; rather we argue that any valid economic model must at least apply to such a situation.

Computer simulation of market models at the level of individual transactions is not entirely new. For example, the role that computer models of economic agents can play in studying economic questions is described by Holland and Miller.<sup>9</sup> Models of markets in which human traders are replaced by computer programs are described by Gode and Sunde,<sup>7</sup> Rust *et al.*<sup>14</sup> and Wellman.<sup>18</sup> The work described here differs from this previous work in that we attempt to construct a model of a *complete* albeit very simple economy. That is, each agent produces a commodity, consumes a commodity, and engages in trading. In this way we can study the interaction among prices, production, consumption, and aggregate supplies and demands along with the effects of speculators.

Our goal is to build (eventually more complicated) models that capture aspects of economic reality based on specific assumptions about human behavior. The approach we take has the important advantage of being flexible enough so that these assumptions can be changed or modified easily. In prin-

ciple, we should be able to simulate a model of a large economic community with thousands of commodities using (hopefully accurate) models for the behavior of individual agents. Such a system could be used to study the effects of government policy, monopolies, and so on. The real problem, of course, is how to construct this model. We are attempting to take a first step in this direction.

The very simplest version of the model, with no prediction or history-based behavior by agents, results in the emergence of large-amplitude endogenous cycles in price and volume traded. This reflects well known properties of systems in which current action is determined by observations of past price, such as the cobweb model.<sup>1</sup> Even at this point, however, it is not clear what aggregated analytical model could be used and analyzed to predict cycle amplitude, frequency, and degree of regularity. The system simulates the actions of 1,000 independent agents unless otherwise noted, each with his own inventory and skills (at producing food and gold), and the agents interact through an auction that establishes a commonly accepted transaction price.

We use this starting point to examine the effects of speculators on price stability and market efficiency. The first result is that speculators, agents who simply try to buy low and sell high, stabilize the price dramatically. The framework then allows us to compare the efficiency of different speculation algorithms, and to study the effects of speculation on overall market efficiency. As we expect, the stabilization of price results in an overall increase in market efficiency and fluidity, in the sense that individual production decisions are more closely matched to skill, and the numeraire is more easily converted into accumulated wealth.

## 2. The Model

### 2.1. Basic Assumptions

Our approach is to build a minimal system that exhibits interesting economic behavior. Such a system cannot possibly consist of only one good, because the concept of price would have no meaning. Therefore, it must contain at least two distinct commodities, which we call *food* and *gold*. We assume agents consume one unit of food during each period.

Each agent is capable of producing both goods, at predetermined rates called *skill levels*. We will denote the skill levels of agent  $i$  by  $skill_f[i]$  and

$skill_g[i]$ . For example, an agent with  $skill_f[i] = 3.21$ , produces 3.21 units of food on a day in which he decides to farm. (Throughout this chapter we will use the word *day* to characterize the indivisible unit time period.) The skill levels of each agent are predetermined by random numbers from a uniform distribution, and are constant for the duration of the simulation. It follows that some agents will be more efficient than others in producing food or gold. In addition, agents have the ability to maintain an inventory of food and gold, at no cost.

Every day each agent must decide whether to produce food or gold. This decision is based on the price at the end of the previous day. Since agents are assumed to be wealth-maximizing, agent  $i$  produces gold during day  $t$  if and only if

$$skill_g[i] > P(t-1) \cdot skill_f[i] \quad (1)$$

where  $P(t-1)$  is the most recent price; that is, if the value of his gold production would exceed the value of his alternative food production.

Agents are also assigned a *reserve level* of food, representing the food inventory which that particular agent wishes to maintain. These reserve levels are predetermined by random numbers from a uniform distribution, just as are skill levels, and also remain constant throughout the simulation. In our experiments, we used 20.0 and 40.0 as lower and upper bounds on reserve levels.

In our minimal economic system there must be a trading mechanism so that agents can buy or sell food in exchange for gold if they so desire. The use of an auction for this purpose emerges naturally. Ability to trade also implies the existence of a *price* of food, in units of gold, which should adjust according to supply and demand. The exact method of price determination will depend on the specific auction mechanism used.

Finally, it is crucial to define *utility* for each agent. Indeed, in order to set bids or offers, agents must act according to some utility function, which captures the degree of their willingness to buy or sell.

## 2.2. The Utility Function

Every period, each agent submits either a bid to buy, or an offer to sell. (For simplicity, we often use the term *bidder* to refer to a buyer or seller, and the term *bid* to refer to a bid to buy or offer to sell.) Each agent's particular bid is determined by multiplying his particular utility function by the price  $P(t-1)$ , which resulted from yesterday's auction, and which is known to everyone. The

utility function of an agent depends on his food and gold inventories. For simplicity we take the utility function to be the same for all agents.

The important characteristics of the utility function are that the bids increase (resp. decrease) when food inventory is low (resp. high). We also want the utility function to increase with gold inventory when bidding to buy, and to decrease with gold inventory when offering to sell. That is, as the bidder gets richer, he is willing to pay more for food when buying, and to sell food for less when he is selling. For the examples considered it was found that the particular shape of the curve is not critical, and the particular function was chosen as follows, parameterized by only three quantities. Let  $\bar{f} = f[i]/r[i]$ , the food inventory of agent  $i$  normalized by his reserve; and  $\bar{g} = g[i]/(P \cdot r[i])$ , the gold inventory normalized by the current value of his reserve. We then choose the values of the bid function  $B(\bar{f}, \bar{g})$  at  $\bar{f} = 0$  and  $\bar{g} = 0, 1, \text{ and } \infty$  as follows:

$$\begin{aligned} B(0, 0) &= b_{00} \\ B(0, 1) &= b_{01} \\ B(0, \infty) &= b_{0\infty} \end{aligned} \tag{2}$$

and define the bid function at  $\bar{f} = 0$  by the following exponential function of  $\bar{g}$ :

$$B(0, \bar{g}) = b_{0\infty} - (b_{0\infty} - b_{00})e^{-\gamma\bar{g}} \tag{3}$$

where

$$\gamma = \ln \left( \frac{b_{0\infty} - b_{00}}{b_{0\infty} - b_{01}} \right) \tag{4}$$

The complete bid function is then taken to be the exponential function of  $\bar{f}$  that passes through the point  $\bar{f} = 1, \bar{g} = 1$ :

$$B(\bar{f}, \bar{g}) = (B(0, \bar{g}))^{(1-\bar{f})} \tag{5}$$

Thus at the point  $\bar{f} = 1$ , which corresponds to the food inventory being exactly at reserve, the bid function is always one, which means that the bid is precisely equal to the current *market price*  $P(t-1)$ . When the food inventory is below reserve, the bid function yields an offer price above  $P(t-1)$ , and when the food

inventory is above it yields an asking price below  $P(t - 1)$ . See Fig. 1 for an illustration. Again, we remark that the details have proven not to be critical, and this function has been chosen for simplicity and transparency. Finally, the amount bid is simply the difference between the agent's current food inventory, and his reserve level.

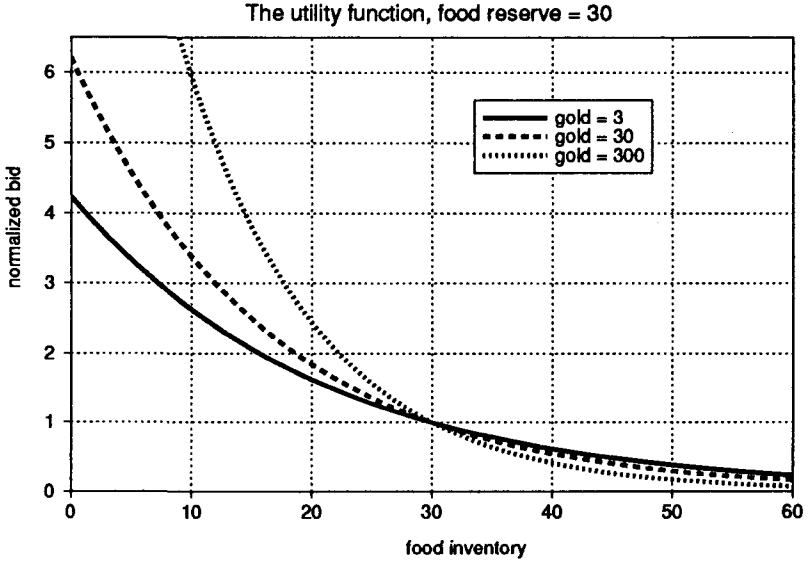


Figure 1. The shape of the utility function, the normalized bid  $B$  vs. food inventory for three values of gold inventory. The particular case shown is for the parameters  $b_{00} = 4$ ,  $b_{01} = 8$ ,  $b_{0\infty} = 16$ ,  $P = 2.0$ , and food reserve of 30.

### 2.3. The Auction

There are many ways in which an auction can operate, and the literature on the topic is voluminous. A comprehensive review can be found in Engelbrecht-Wiggans *et al.*<sup>4</sup> For the purpose of this model we considered two forms of auctions. The first is a *sealed-bid* auction, such as the one attributed to Martin Shubik, and described in Smith *et al.*<sup>15</sup> The advantage of this auction is that it leads to a clearly defined procedure with few arbitrary choices. A central auctioneer collects bids and offers from agents, and determines the market-

clearing price. Subsequently, all transactions occur at this price. An example of sealed-bid auctions are call markets which set the price to maximize the amount traded, as reported by Harris.<sup>8</sup>

The second form is a *double auction*, where buyers and sellers are matched up in some way, and trade at different prices. In the most common type of double auction the highest bid to buy and the lowest offer to sell “hold” the market. Participants may then either raise the current highest bid, lower the current lowest offer, or accept one of the two. This process is described in Friedman<sup>5</sup> in the context of financial markets, and was actually implemented by Williams and Smith<sup>20</sup> in laboratory experiments. For some examples of the theoretical literature on double auction models, we refer the reader to Williams<sup>21</sup> and Friedman.<sup>6</sup> Another type of double auction is characterized by buyers and sellers matching up either randomly, or in some other distributed fashion (see Wolinsky<sup>22</sup>).

The majority of our experiments assume a sealed-bid auction, in which an intelligent auctioneer finds and announces the unique market-clearing price for the day. The auctioneer receives two lists each day: a list of bids to buy, along with the amounts desired, plus a list of offers to sell and the corresponding amounts available. This process of communication between agents and the auctioneer is described in Hurwicz<sup>11</sup> using the concept of “languages.” In the case of a Walrasian tâtonnement process, “messages are the proposed prices and commodity bundles.” Later we describe experiments which assume a distributed double auction.

For a reasonable market, we must first ensure that the following constraints are satisfied:

- (a) No buyer pays more than his bid.
- (b) No seller sells for less than his offer.

Next, as mentioned above, we assume that the auctioneer operates to maximize the total quantity of food traded. One motivation for this is the fact that an auctioneer is likely to be rewarded for high sales volume, but the auctioneer does not own and trade commodities in this model. From the point of view of the agents, this strategy attempts to satisfy the maximum number of participants in the auction. We enforce a third constraint:

- (c) The maximum amount of food is traded, subject to constraints (a) and (b).

These three constraints determine a range of possible prices each day. To see this, consider for each price  $P$  the amount that would be available for sale at that price, and the corresponding amount that would be bought, as shown in Fig. 2. For purposes of discussion, the figure shows the case where total demand exceeds total supply on the day of the auction, but the following comments with appropriate changes also apply to the case where supply exceeds demand. We call these curves *supply volume* and *demand volume* respectively, and emphasize that they change from day to day with inventories and price. The supply and demand curves associated with a particular auction are monotonically non-decreasing and non-increasing, respectively, and are piecewise constant with discontinuities at prices corresponding to the bids of agents. The closed interval of prices  $[A, C]$  in Fig. 2 represents the range of prices that satisfy Constraints (a)–(c). At prices below  $A$  total amount of food for sale is less than aggregate demand; at prices above  $C$  total amount of food for sale is greater than aggregate demand. Further, in the range  $[A, B]$  there is more than one buyer willing to meet the price, so it is only in the semi-open interval

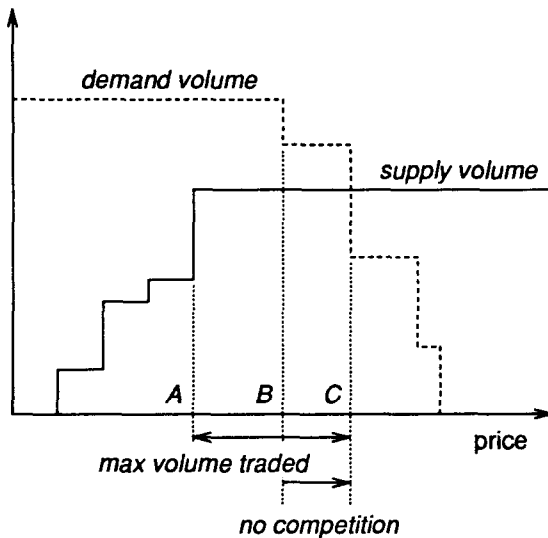


Figure 2. Illustration of the sealed-bid auction. Price is determined by maximizing volume. This maximum volume is achieved for prices in the closed interval  $[A, C]$ , whereas in the semi-open interval  $(B, C]$  the Bidder  $C$  has no competition.

$(B, C]$  that the set of buyers and sellers is determined uniquely. That is, in  $[A, B]$  there is competition between the buyer whose bid is  $B$  and the buyer whose bid is  $C$ , which is resolved by raising the price above  $B$ .

The choice of a single price in the range  $(B, C]$  is somewhat arbitrary, and we choose the upper limit  $C$ . This has the effect of raising the price fastest when demand exceeds supply in a particular auction. That is, when demand exceeds supply we choose the highest price consistent with Constraints (a)–(c). Symmetrically, when supply exceeds demand we choose the lowest price consistent with the constraints. In practice these choices are not critical because in a simulation with many agents, the piecewise-constant intervals are quite small (see Fig. 3). If maximum supply should equal maximum demand precisely, we set the price equal to that of the preceding day.

To resolve the remaining special cases, when the supply is zero but the demand is not zero, the new price is set equal to the highest bid. When the demand is zero but the supply is not zero, the new price is set equal to the lowest asking price.

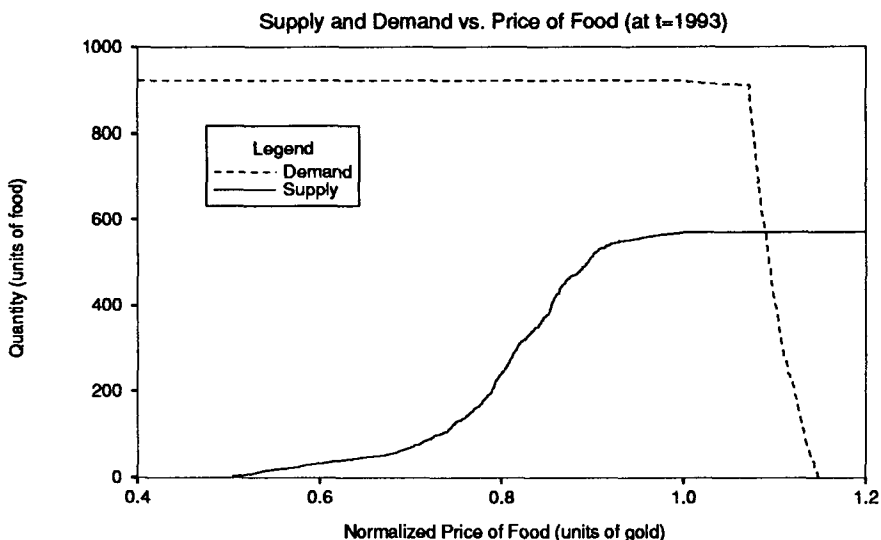


Figure 3. Price determination in the auction for an example with 1,000 agents. Maximum volume is traded subject to the bid/ask constraints. The price increases when demand exceeds supply, and vice versa.

## 2.4. Speculators

Our model includes two different classes of speculators, who differ mainly in the trading rule they use. Speculators are not directly engaged in production activities, do not consume food, and start the simulation with a fixed inventory of gold. Speculators in both classes offer their entire inventory of food for sale when they sell, and try to purchase as much as they can afford when they buy.

The first class of speculator (named DER) uses a trading rule based on the estimated second derivative of the price curve. They post a bid to buy when the slope of the price curve is increasing, and an offer to sell when the slope is decreasing. If speculator  $j$  decides to buy, he posts a bid of  $P(t-1) \cdot (1 + \text{margin}[j])$ . Conversely, if he decides to sell, the offer is  $P(t-1) \cdot (1 - \text{margin}[j])$ , where  $\text{margin}[j]$  is specific to each individual speculator. In this way, DER speculators can profit from price fluctuations, because of the additional information they possess on how the price is changing.

Speculators in the second class, which we name AVG, use adaptive expectations to predict the average price. Adaptive expectations are described in Carlson (1967), in reference to the cobweb model. The price forecast is given by

$$\hat{P}(t) = \beta \cdot \hat{P}(t-1) + (1 - \beta) \cdot P(t-1) \quad (6)$$

where  $\hat{P}(t)$  is the expected price at time  $t$ , and  $\beta$  is a weighting coefficient. A decision to buy is made by speculator  $j$  when

$$P(t-1) < \hat{P}(t) \cdot (1 - \text{margin}[j]) \quad (7)$$

in which case a bid of  $P(t-1) \cdot (1 + \text{margin}[j])$  is posted. The use of the same margin to trigger trade decisions and to set the bid is arbitrary, but adopted for simplicity. The logic is quite simple: if the previous period's price is sufficiently below the forecasted average, then the speculator should buy. Conversely, speculator  $j$  sells when

$$P(t-1) > \hat{P}(t) \cdot (1 + \text{margin}[j]) \quad (8)$$

in which case an offer of  $P(t-1) \cdot (1 - \text{margin}[j])$  is posted.

### 2.5. Competitive Equilibrium Price

Within this model, the long-run competitive *equilibrium price* can be computed at the beginning of the simulation. It is simply the price at which just enough agents produce food to satisfy the needs of all nonspeculating agents.

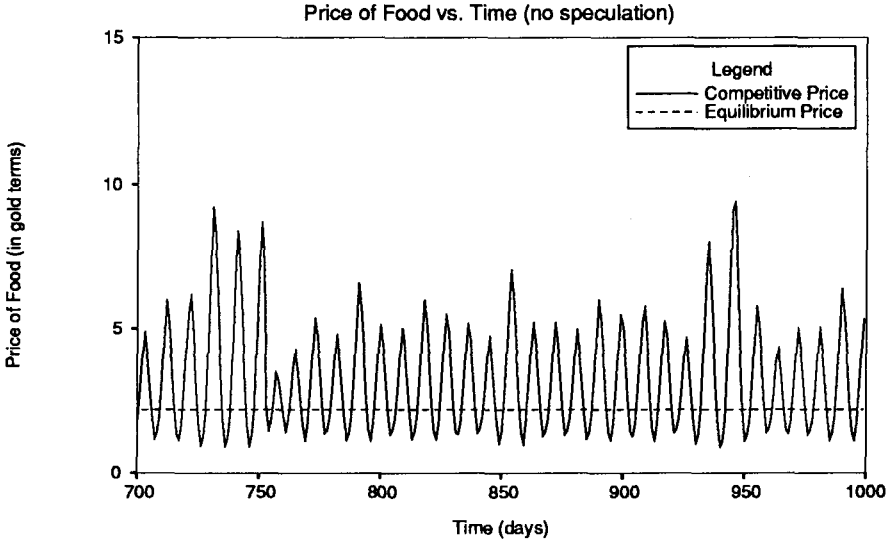


Figure 4. Price of food vs. time, in the absence of speculation. The price curve displays endogenous oscillations. The equilibrium price is represented by the dashed line.

### 3. Simulation Results without Speculators

In the absence of speculation, the price history is oscillatory with an amplitude that depends on the simulation parameters (see Fig. 4). These price movements can be explained by the fact that the market reacts to excess supply or demand by “overshooting” the competitive equilibrium level. As the price increases, agents start shifting from the production of gold to the production of food, which is becoming more profitable. Eventually, food accumulates in the system, due to overproduction, leading to sell orders in the auction, and driving the price down again. Experiments show that the magnitude of price oscillation is reduced if the agents use memory or prediction. Rather than pursue

that path we choose to incorporate forecasting by introducing speculators, as described in the next section.

In systems without speculation, the volume traded also oscillates considerably (see Fig. 5). This is a direct consequence of the price fluctuations, which determine the actions of the agents. From the figure it is clear that the volume traded is highest at price turning points. This is due to the fact that these turning points represent close matching of supply and demand (see Fig. 6), which leads to more trade. When one side of the market is short, the volume traded is correspondingly low.

In the model without speculation, the market's behavior seems reasonable. Agents stay as close as possible to their reserve levels of food, and the price oscillates considerably, due to the inertia in market adjustment to excess demand (or supply).

We now turn to another interesting question: what factor is primarily responsible for an individual agent's net worth? We use gold inventory at the end of the simulation as a measure of net worth. This seems reasonable, since each agent attempts to accumulate gold. Figure 7 shows net worth as

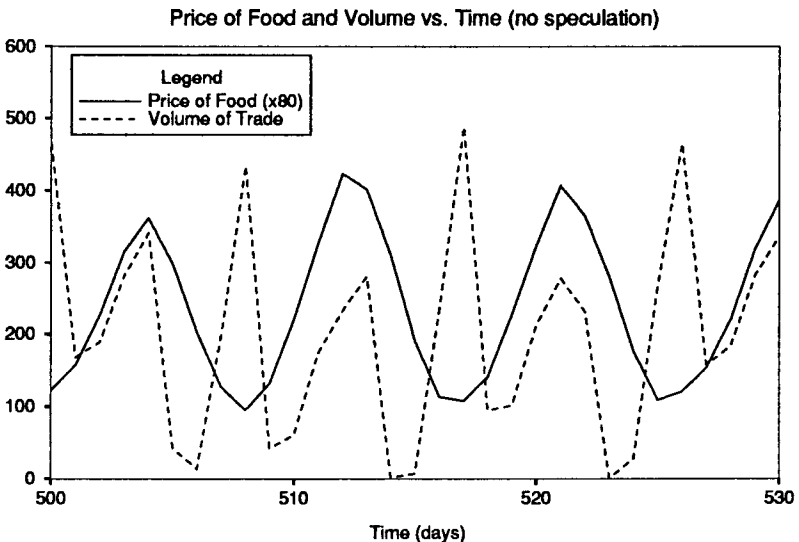


Figure 5. Volume traded and price of food vs. time, in the absence of speculation. Periods of high volume occur at price turning points.

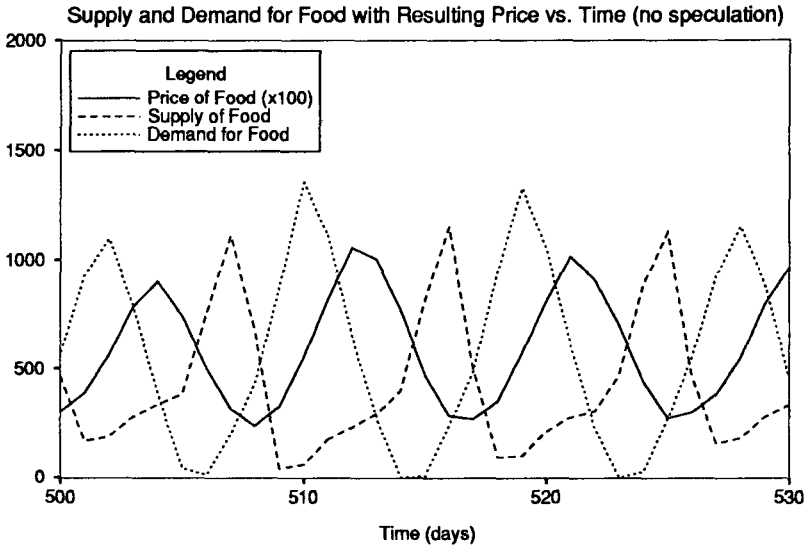


Figure 6. Supply and demand in the auction, and market-clearing price vs. time, in the absence of speculation. Turning points in the price curve correspond to “well-balanced” markets (supply and demand match).

a function of agent food and gold skill. Note that food and gold skills are given in normalized terms, in the sense that a skill level of “0” means that the corresponding agent is at the lowest end of the skill spectrum, whereas a skill level of “1” means that the corresponding agent is at the highest end of the skill spectrum. It appears that there exists a strong correlation between skill in the production of food, and ultimate wealth. In contrast, gold skills seem to have no significant influence on final net worth. Also, net worth increases in an approximately linear fashion with food skill between 0.2 and 1, but agents with skill levels below 0.2 do not accumulate wealth. What appears to happen is that agents with relatively high gold skill tend to mine more readily than those with lower gold skill, with the expectation of buying food at the price when the mine/farm decision is made. But on the average purchases of food must be made at higher prices. It is clear that lack of foresight puts agents with a high temptation to mine at a relative disadvantage to those who are more inclined to farm.

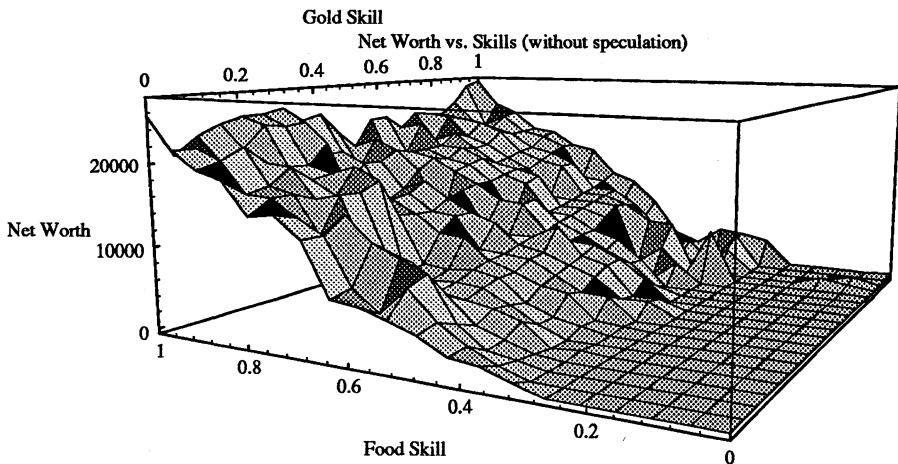


Figure 7. Net worth as a function of food and gold skills, in the absence of speculation. The data represents 10,000 agents after 5,000 days of simulation. Food and gold skills are normalized so that the maximum value is 1.

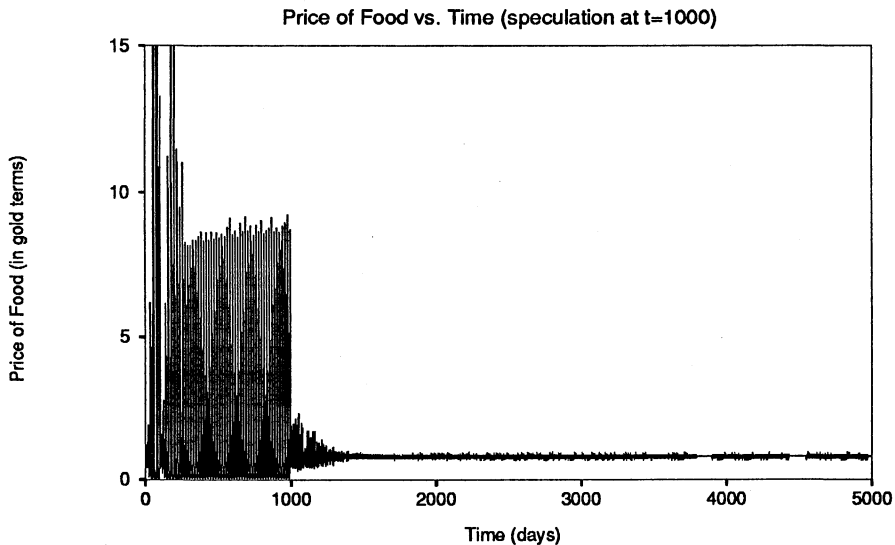


Figure 8. Price of food over time, with the introduction of 100 speculators of each class at  $t = 1,000$ . The introduction of speculation greatly dampens price fluctuations.

#### 4. Simulation Results with Speculators

Williams and Smith<sup>20</sup> report that introducing speculators in laboratory experiments on double auctions reduces price fluctuations, and we observe the same effect in our system. Figure 8 shows the results of introducing speculators of both classes at  $t = 1,000$  days. The stabilization is dramatic. In fact, after introduction of speculators the price stays very close to the equilibrium value, oscillating generally within a range of 5–10% (see the detail in Fig. 9). Price movements also become less regular.

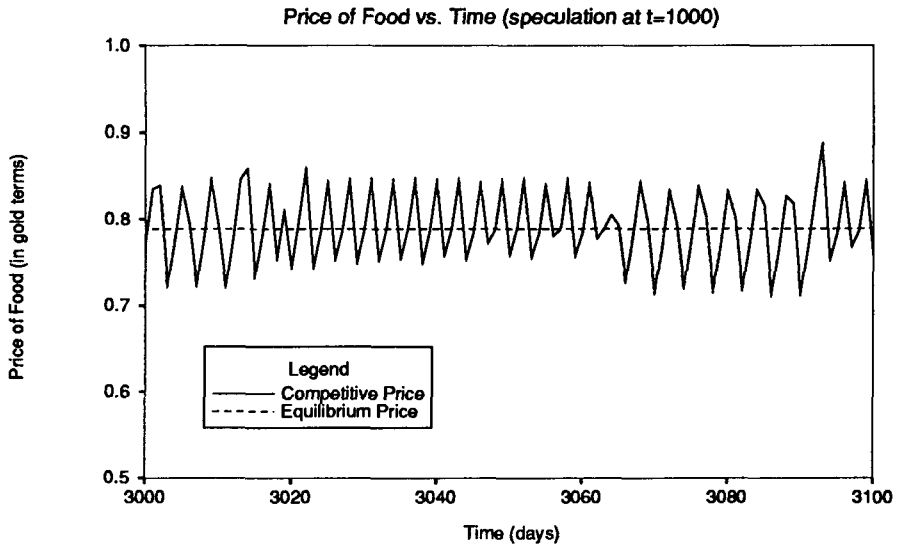


Figure 9. Price of food over time, with speculation. Prices tend to oscillate within 5–10% of the equilibrium value.

We observe that speculators also have a stabilizing effect on the volume of trade (see Fig. 10). Specifically, speculative activity maintains a certain minimum level of trade, close to 800 units of food per day, whereas the volume of trade regularly dips to zero or near zero in the absence of speculation (see Fig. 5). This clearly demonstrates that speculation has made the market more fluid.

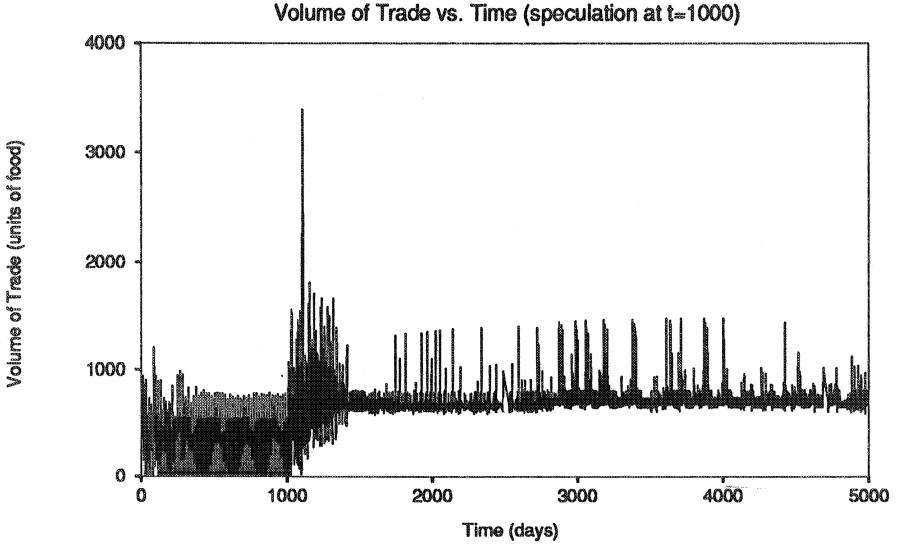


Figure 10. Volume of trade over time, with speculation at  $t = 1,000$ . Speculative activity has made the market more fluid.

We define the concept of *Gross Domestic Product* within our system as the total gold-equivalent value of productive output. Since nominal GDP is subject to price fluctuations, we define *real GDP* as constant-unit GDP, using the equilibrium price of food  $P_e$ . Thus, at any given point in time  $t$ , real GDP is defined in a system of  $n$  agents as

$$GDP = \sum_{i=1}^n stock_g[i] + P_e \cdot \sum_{i=1}^n stock_f[i] \quad (9)$$

where  $stock_g[i]$  and  $stock_f[i]$  are gold and food inventories of agent  $i$ , respectively.

It appears that real GDP grows linearly, with a discontinuous change in slope at  $t = 1,000$  corresponding to the introduction of speculators (see solid line in Fig. 11). The dashed line on the graph represents GDP growth, in the absence of speculation. This result shows that speculation increases the efficiency of the market, since GDP (which is a measure of aggregate wealth) grows more rapidly after speculators are introduced. Williams and Smith<sup>20</sup> come to the same conclusion in their experimental market:

Using observations from two cyclical market designs, we have shown that the inclusion of a class of speculative agents tends to reduce significantly the observed magnitude of cyclical price swings relative to those observed in markets without intertemporal speculation. Including speculators also results in a significant increase in market efficiency.

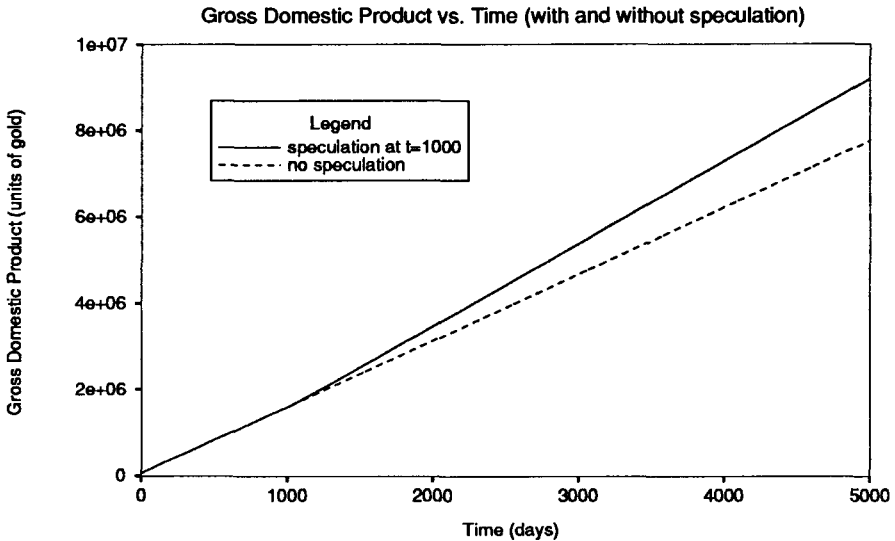


Figure 11. Constant-unit Gross Domestic Product, with speculation at  $t = 1,000$ . The dashed line represents the GDP in the absence of speculation. Increased slope of the GDP curve after the introduction of speculators, relative to the dashed line, is a sign that the market is more efficient with speculation.

The GDP discussed here includes the wealth accumulated by speculators, but if we were to plot the GDP of the productive agents only, the difference would not be discernible. The speculators do not absorb a significant fraction of the extra wealth.

We can now investigate net worth as a function of skills, as we did in Section 3, where we discussed a system without speculation. Figure 12 displays final net worth as a function of food and gold skill, and shows clearly that the relation is now well described as the intersection of two planes. If we consider

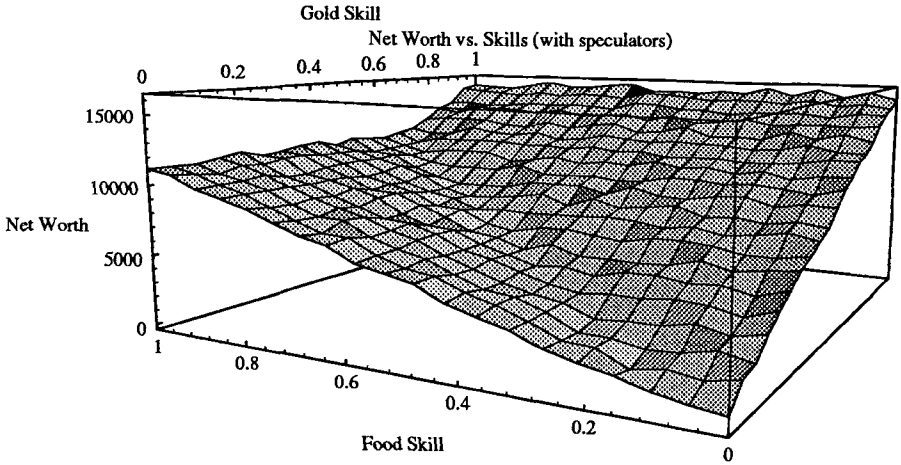


Figure 12. Net worth as a function of food and gold skill, in system containing speculation. The data represents 10,000 agents after 5,000 days of simulation. Food and gold skills are given here in normalized terms.

a particular food skill level, say 0.6 (in normalized terms), and scan along the gold skill axis in the increasing direction, there are two distinct segments: first a segment of constant net worth, and then a segment of increasing net worth. The transition point is that gold skill at which agents with the particular food skill of 0.6 switch to the production of gold, and thereby accumulate greater net worth. A similar argument can be made if we scan along a given iso-gold-skill line in the direction of increasing food skill. The intersection of the two planes is in fact determined by the condition that a day's production of gold buys a day's production of food at the equilibrium price; in other words,  $skill_f \cdot P_e = skill_g$ .

The economic interpretation of this phenomenon is straightforward. More stable prices allow agents to specialize in their good of comparative advantage: agents who are highly skilled miners now find it easier to accumulate gold while trading enough gold for food to maintain desired inventories.

## 5. Robustness of the Model

Until this point, we have introduced a market model based on very specific assumptions, and described its behavior. It could be argued that the results

obtained from our system are not sufficiently general in scope, since many seemingly arbitrary decisions were made in the model design process. We now attempt to demonstrate that the results are insensitive to what is perhaps the most critical modeling decision, the choice of auction.

Although most of the experiments were conducted using a sealed-bid auction, a system was also simulated containing a distributed double auction. Specifically, buyers and sellers are matched up in the following manner: first the seller with the lowest offering price is allowed to trade with the buyer bidding highest. When one of the agents has been satisfied, the next lowest seller (or next highest buyer) is allowed to trade. This process continues until either the buyer or seller list is exhausted. The price of each individual transaction is determined as the midpoint between the offer and bid prices. This means that in general each transaction occurs at a different price. We define  $P_1$  as the average of these individual transaction prices, weighted by the volume traded at that price. That is,

$$P_1 = \frac{\sum_{i=1}^m P_i \cdot Q_i}{\sum_{i=1}^m Q_i} \quad (10)$$

where  $P_i$  and  $Q_i$  are respectively the price and quantity associated with transaction  $i$ , and  $m$  is the number of transactions. An alternative definition of the daily price would simply be the average of bids posted for the day, weighted by the desired quantity of food traded associated with that bid. This procedure defines a price  $P_2$  in a way that does not depend on transactions that actually take place. In a model containing  $n$  agents, the daily bid of agent  $i$  is denoted as  $bid[i]$ , with an associated quantity  $qty[i]$ . Then by definition

$$P_2 = \frac{\sum_{i=1}^n bid[i] \cdot qty[i]}{\sum_{i=1}^n qty[i]} \quad (11)$$

Clearly,  $P_1$  is an accurate measure of the “real” price of food, if the volume actually traded is high. However, on low-volume days, there may not be enough transactions to make  $P_1$  a meaningful price; on such days,  $P_2$  becomes a more reasonable estimate of the value of food. This leads to the following price

definition:

$$P' = \alpha \cdot P_1 + (1 - \alpha) \cdot P_2 \quad (12)$$

where  $\alpha$  represents the fraction of agents who engage in trade on any given day.

Using this model, we obtain results similar to those in the sealed-bid situation (see Fig. 13). Other price-setting mechanisms were also tested, and did not significantly alter the system's behavior. The general issue of the influence of price-estimation algorithms on the behavior of distributed auctions is an interesting topic for further research.

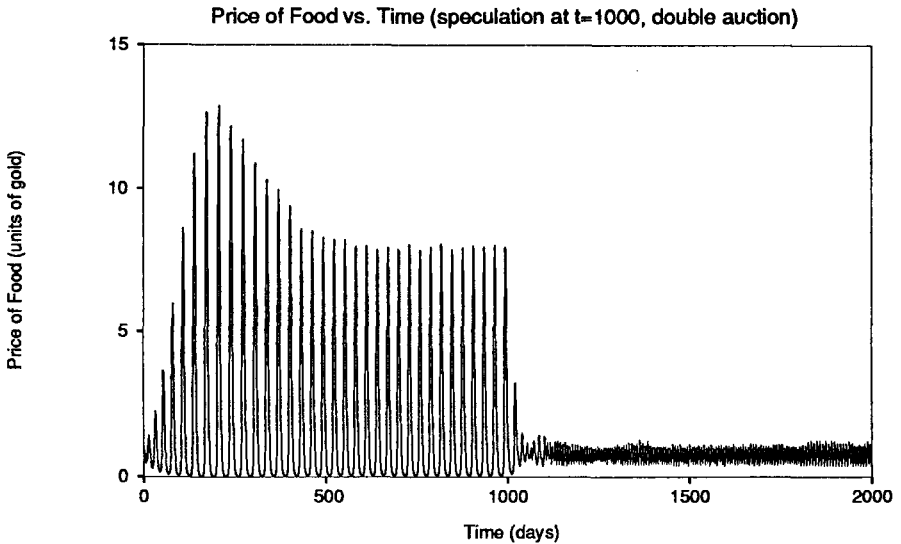


Figure 13. Price of food over time, with the introduction of 100 speculators of each class at  $t = 1,000$ . In this experiment, a distributed double auction mechanism was used. Note that price movements are similar to those in Fig. 8.

## 6. Performance of Speculation Rules

An interesting issue is how well each class of speculators performs in our system. For this purpose, we experimented with systems containing only DER speculators, only AVG speculators, and finally both together.

Figure 14 represents DER speculator aggregate wealth over time. Note that for speculator  $j$ , wealth is defined as  $stock_g[j] + P_e \cdot stock_f[j]$ , where  $P_e$  is the equilibrium price in the system, as defined earlier. From Figs. 8 and 13 it is clear that the trading rule based on changes in the estimated derivative of the price curve is quite profitable, in the absence of other types of speculators.

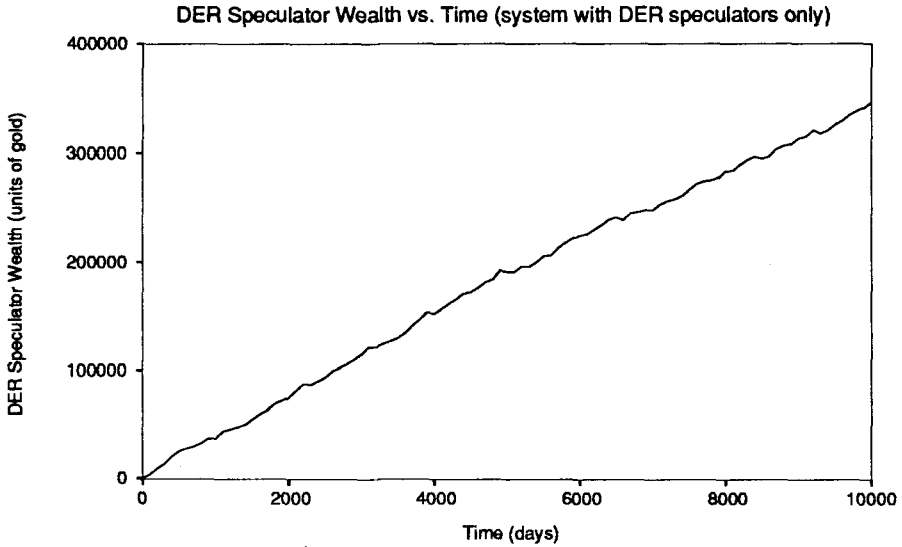


Figure 14. DER speculator wealth over time (in a system where DER speculators operate alone).

AVG speculators also become increasingly wealthy when they operate alone in the market (see Fig. 15). However, they do not accumulate as much aggregate wealth as do the DER speculators in Fig. 14. In the case corresponding to Fig. 15 (for AVG speculators) prices oscillate more closely around the equilibrium level, and that considerably reduces opportunities to make arbitrage profits.

We turn next to the most interesting case: DER and AVG speculators operating within the same system. Figures 16 and 17 show how both classes of speculators perform. DER speculators do not generate much profit: they rapidly lose the gold they were credited with at simulation start, and do not accumulate more than 100 units of gold for the remainder of the simulation.

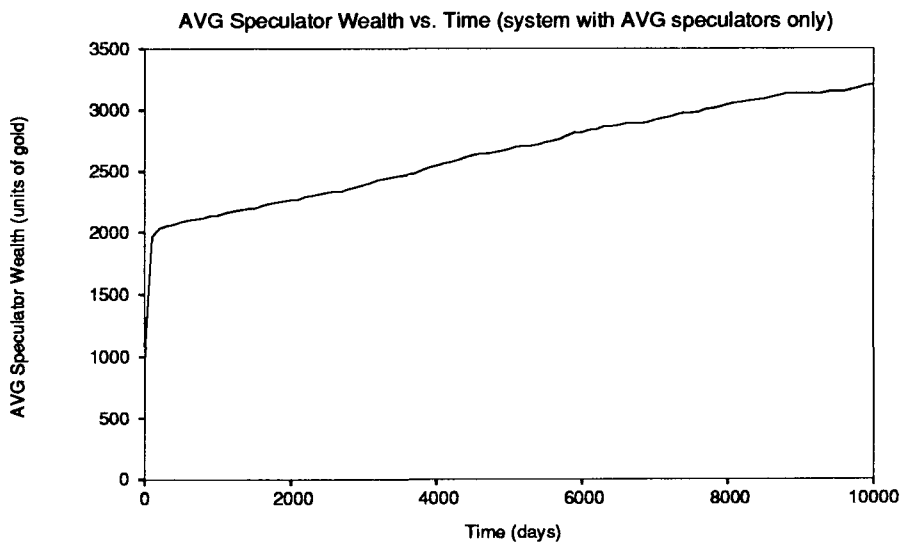


Figure 15. AVG speculator wealth over time (in a system where AVG speculators operate alone).

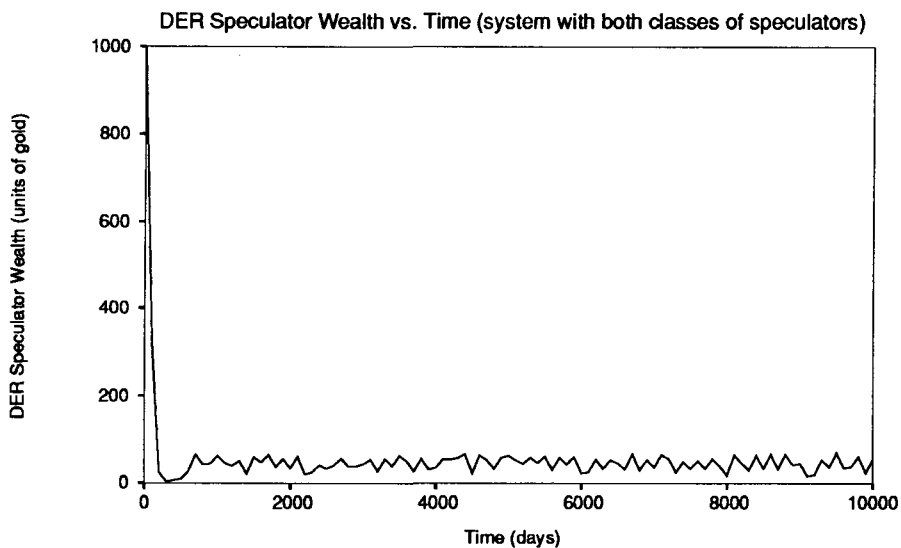


Figure 16. DER speculator wealth over time (in a system containing both classes of speculators).

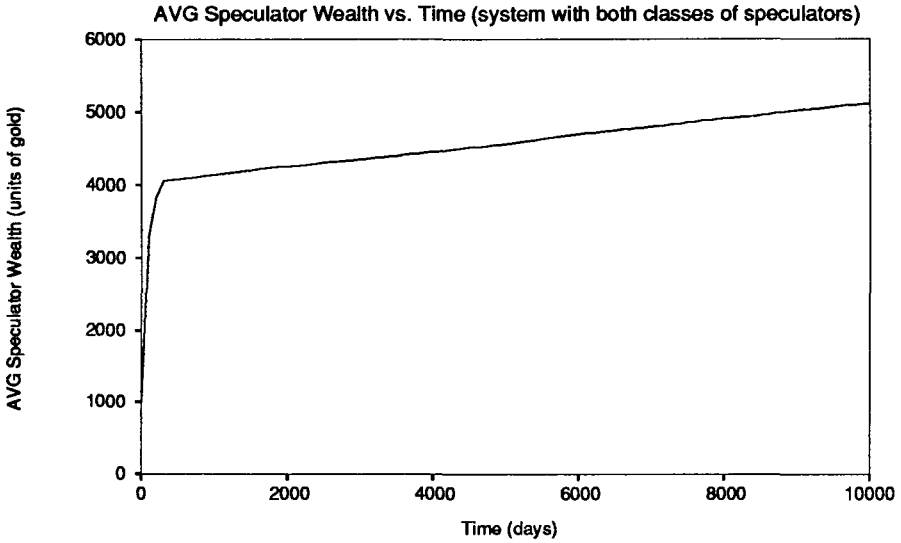


Figure 17. AVG speculator wealth over time (in a system containing both classes of speculators).

In contrast, AVG speculators perform consistently well (Fig. 17). In the early phase of the simulation, there seems to be a transfer of wealth from DER to AVG speculators, as reflected by a sharp rise in AVG speculator wealth as DER wealth drops rapidly.

These results indicate that DER speculators need large price fluctuations. When AVG speculators are added, the magnitude of price oscillations is greatly reduced, ruining all DER speculators except those with the smallest trading margins.

Finally, we examine how wealth is distributed among AVG speculators, in a system where both classes of speculators are present. In particular, Fig. 18 shows a plot of speculator wealth vs. the margin assigned to each speculator. Speculators with higher margins tend not to make as much profit as those with lower margins. This is caused by two factors. First, speculators with high margins trade only when there is a large gap between the forecasted and actual prices. Second, speculators with high margins submit bids to buy that are lower than those of their counterparts with low margins, and offers to sell that are higher, and the auction algorithm gives priority to more competitive

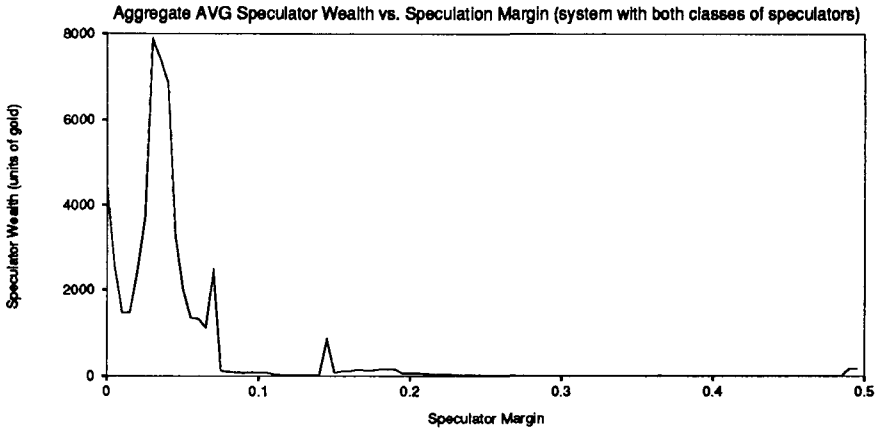


Figure 18. AVG speculator wealth as a function of speculation margin.

bids. If the other side of the market is not large enough, it can happen that speculators with high margins do not trade at all, which explains their lack of success.

## 7. Conclusions and Discussion

Can the dynamics that emerge in our system be analyzed within a manageable theoretical framework? A clear candidate for such a framework is the cobweb model and its variations. Unfortunately, it does not appear that cobweb theory can add to our understanding of the behavior of our system, mainly because supply and demand functions change daily, as agents update their production decisions. The underlying assumptions of the cobweb model are simply not satisfied by our system. The introduction of speculation further complicates any attempt to analyze the system theoretically. Future work with this approach can introduce even more complicated mechanisms, and we view the simulation as an experimental testbed, rather than a fixed structure which is susceptible to mathematical analysis.

Economists have observed chaotic or other complex dynamic behavior in simple models. For example, Chiarella<sup>2</sup> shows how chaotic behavior can emerge in a cobweb model assuming a fairly general non-linear supply curve. Our system is more complicated than his, and so the irregular behavior illustrated

in graphs like Fig. 10 is not surprising. We have in fact observed boundaries between sets of skill parameters that demark periodic and seemingly chaotic behavior. This subject is worthy of further study, but has not been addressed in this chapter. The recent book edited by Creedy and Martin<sup>3</sup> provides introductory material and presents some recent work on chaos in economics.

Enormous computational resources are becoming available at very low cost, and we hope the approach described in this chapter is a start towards using such resources to gain insight into real economic problems. In fact the program we describe barely begins to use even what is available today; a 1,000-day simulation of 1,000 agents and 200 speculators runs in 73 seconds on a DEC 5000 workstation. We view simulation at the agent level as a third way to study the economic consequences of individual behavior, complementing theory and experimentation with human subjects.

The succeeding chapters of this book deal with a variety of ways to use market mechanisms for distributed resource allocation. The simulations we have described, apart from providing a tool for general economic inquiry, tend to confirm the following generally accepted characteristics of such systems:

- Competitive price determination can be used to “control” or “stabilize” price, volume, and other system variables;
- Still, for certain systems at least, there may be large fluctuations that look quasi-periodic, and have chaotic characteristics;
- Artificial speculators (also called traders) can further stabilize price to a great extent, leaving a much smaller residual component, and resulting in more effective resource allocation.

Oscillations without speculators may be large in amplitude, and it may be that certain parameters must be tuned to achieve nicely damped behavior. This behavior is closely analogous to that of classical feedback control systems. For example, in the energy control system described in Clearwater *et al.* in Chapter 10, an inappropriate gain can cause the system to oscillate in response to a sudden change in sunlight distribution on the building. This suggests trying to use artificial traders to stabilize behavior further or eliminate the need to hand-tune parameters.

## Acknowledgment

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