

# I INTRODUCTION

The subject matter of this book is Non-relativistic Quantum Mechanics, a theory which, needless to say, explains an enormous wealth of observations. This fact by itself would constitute a sufficient reason to study the theory in some depth. But there are also other reasons. Non-relativistic Quantum Mechanics, which we shall simply call Quantum Mechanics, rests on a sound mathematical basis which means that the basic concepts of the theory can be understood precisely. This precise understanding is essential to understand and to contribute to the development of two broad fields at the limits of our present knowledge. One is the relativistic theory of quantum fields which constitutes today one of the frontiers of theoretical physics. The other, that lies at the boundary between physics and philosophy, is the understanding of the relation between ourselves and the world that we study.

While the physical observations are non-mathematical entities ultimately accessible to our senses, the theory introduces some fundamental concepts that are mathematical and cannot be explained by appealing to our sensory experience. As a consequence it is not possible to understand Quantum Mechanics without understanding the mathematical theory in which those concepts arise.

It is the purpose of this chapter to explain in some detail the brief remarks of the previous paragraph.

## 1. Description of experiments

There are of course many reasons that may lead physicists to conceive of an experiment. One may want to test a prediction of a more or less well developed theory. Another may be seeking some orientation in the development of a tentative theoretical formulation. Still another may want to determine the properties of some material with an eye to possible applications. Whatever the motivation, in

most if not in all cases, the experimenter has in mind some more or less elaborated theoretical framework to guide him in the design of the experiment.

Once the design has been completed the experimenter decides what instruments are needed, proceeds to draft them in detail, and has them built in the technical shop.<sup>1</sup> The important point to be made here is that the detailed plans of the instruments are completely understandable to the shop staff even if they know nothing about the theoretical ideas that guided the design.

After the experimental instruments have been built and assembled, the experiment, which may consist of one or more experimental runs, is performed.

At the start of an experimental run some of the instruments are adjusted by manipulating knobs or other devices until their settings, as verified by the readings of appropriate dials, are the desired ones. Some time later the run is completed by noting another set of dial readings which constitutes the experimental results of the run.

When the predictions of a theory are statistical, as is the case in quantum mechanics, an experiment consists usually of many runs with identical initial settings of the instruments. Although normally these runs are performed in succession, we may think of them as starting simultaneously and using identical copies of the experimental setup.

A proper account of the experiment can be phrased in plain language in the sense that it need not contain any reference to the theoretical ideas that entered in its conception; all that is required is a description of the experimental instruments, the instrumental settings at the start, and the dial readings at the completion of each run. As already remarked, the instruments can be built, and therefore described, without any use of those theoretical ideas, while the readings of the different instruments consist merely of a set of numbers that refer to some variable characteristics incorporated in the construction of those instruments. Of course, in reporting the experiment, the experimenter usually discusses the theoretical ideas that led to its conception and the significance of the results in relation to those ideas but this part of the report constitutes an *interpretation* of the experiment, not an *account* of it.

In practice it is generally true that the instructions for the construction of the experimental instruments are phrased in plain language. On the other hand, in an article intended to describe an experiment to other workers in the field one often finds sentences such as "the proton beam was focused on the target" which

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<sup>1</sup> Some instruments may, of course, have been designed by somebody else and be available from the shelf but this and other such details are clearly irrelevant for our purposes.

is certainly not expressed in plain language, for it contains an obvious reference to the theoretical ideas used in designing the experiment. That sentence, however, could have been replaced by something like “the current in the coils labelled A in the diagram was adjusted until a maximum signal was obtained in the detector”. Another experimenter will know that that was precisely what was done and what he would have to do if he wanted to repeat the experiment. The original sentence is expressed in a sort of shorthand which is based on shared theoretical ideas. Such a shorthand is extremely useful and even essential, for without it communication among physicists would be unbearably slow, but it should be used with care if one wants to avoid ambiguities. Indeed the history of the development of Quantum Mechanics shows that its uncritical use can lead to apparent paradoxes and conceptual errors.

## 2. The concept of physical system

In analyzing an experiment which can be satisfactorily explained by classical physics, it is not the experimental instruments which are stressed in the analysis but rather the *system* on which the experiment is done. This is so because the concept of system as some physical object separate from the other instruments used in the experimental setup is easy to define precisely.

To illustrate the last remark let us look at a simple example from classical mechanics. Imagine an experiment designed to determine the range of an artillery piece. In each run one measures the velocity of the shell when it leaves the nozzle as well as the horizontal distance that it travels. To analyze the experiment one may choose the shell as “the system”. The *state of the system* at any time is given by the values at that time of an appropriate set of coordinates and velocities of the shell. We say that the theory of classical mechanics (including in this case aerodynamics) explains the experiment because it permits us to write a closed system of equations that describes the time evolution of the state of the shell in accordance with the experimental results.

There are three important properties of a classical mechanical system that need to be stressed here.

(i) The system (in our example, the shell) is a part of the experimental setup, constructed as a piece separate from the other experimental instruments, and can therefore be described precisely in plain language.

(ii) The state of the system can be determined, at least in principle, at any time (for example by stroboscopic photography) without altering the predicted states at future times.

(iii) The state uniquely determines the value of every mechanical property (that is, every dynamical variable) of the system.

The precise definition of a classical mechanical system requires, first of all, that a complete set of coordinates be specified, in the sense that prescriptions must be given on how to measure them. It is, of course, possible to define the complete set of coordinates in different ways but the sets so defined are all equivalent because their coordinates are functions of the coordinates of any particular set. The characterization of the system is completed by giving certain intrinsic (that is, pertaining to the system, not to the rest of the experimental setup) parameters, such as masses or moments of inertia.

Systems that share the same complete set of coordinates will be said to belong to the same *type*. Thus the shell of the above example belongs to the type of rigid bodies. But for two shells to constitute identical systems they must have the same moment of inertia and geometrical shape (because the aerodynamic forces depend on the latter). In general we call two systems identical if they belong to the same type and have the same intrinsic parameters.

A parenthetical remark about nomenclature is in order here. In the above description we have implicitly assumed that one characteristic of a system is that its state at any given time uniquely determines, through a *closed* system of equations of motion, the state at any future time. A part of a classical mechanical system can also be assigned a state which is a subset of the set of coordinates and velocities that constitute the state of the system. Although such a part should properly be called a *subsystem*, the prefix is often omitted and the word "system" is loosely applied to it. As we know, the state of a subsystem at a given time does not, in general, determine the state at future times, for the motion may depend on the values of the remaining coordinates and velocities.

There are theories in classical physics in which a system can be defined which does not have property (i) above. An example is classical electromagnetism. In many cases it is convenient in this theory to regard the electromagnetic field as being "the system", although it is obviously not a piece of equipment that can be constructed in the shop separately from the rest of the experimental instruments. The field can be thought of as a physical agent which is produced by some of the experimental instruments and causes the observed correlations between measurements. However, within the domain of applicability of classical physics, it is possible, even in this case, to regard the state of the system, i.e., the field, as given by a set, albeit infinite, of generalized coordinates and velocities such that properties (ii) and (iii) still hold.<sup>2</sup> Thus, the field can be treated as a kind of mechanical

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<sup>2</sup> Admittedly property (ii) requires the introduction of idealized test bodies with vanishingly small charges.

system with an infinite number of degrees of freedom which is again distinct from the rest of the experimental setup.

The concept of *system* in quantum mechanics is much more difficult to define than in classical mechanics. As in classical electromagnetism, property (i) above is not satisfied: it is in general not possible to identify a particular piece of equipment constructed in the shop as being “the system”. Moreover, contrary to the situation in classical electromagnetism, experience tells us that one can not define a system and its state in such a way that properties (ii) and (iii) are obeyed. As a result, the concept of system in quantum mechanics cannot be introduced as a straightforward generalization of the corresponding classical concept.

Rather than basing the presentation in this book on the difficult concept of quantum mechanical system, I have found it preferable to take the concepts of preparation and measurement, which will be discussed Section 3, as the primary ones. Nevertheless the idea of system as the physical agent produced at the start of an experiment that causes the readings of the measuring devices, is extremely useful in quantum mechanics. Because we are taking the concept of system as secondary, a precise definition is not needed here and shall be given in due time. For the moment it is sufficient to state that the definition is somewhat analogous to the one used in classical mechanics: the type to which a system belongs is given by specifying a particular complete set of physical attributes which can be regarded as taking the place of the classical coordinates. But, while in classical mechanics different sets of coordinates are equivalent, in quantum mechanics there are many different complete sets of attributes that are not. As in classical mechanics, a particular system within a type is defined by appropriate intrinsic parameters, and to every experimental arrangement there corresponds a hamiltonian which is a function of appropriately chosen inequivalent complete sets of physical attributes that take the place of the coordinates and momenta.

If one were to conform rigidly to the approach used in this book, one should refrain from referring to a “system” until the concept is defined and describe, instead, the appropriate preparation procedure. This would result in a rather lengthy exposition which we can fortunately forego since the reader is assumed to be already acquainted with quantum mechanics. For example the sentence, “a measurement is performed in an experiment in which the preparing instrument consists of an oven filled with helium and equipped with a collimator” will be shortened to “a measurement is performed on a beam of helium atoms”. This is an example of the shorthand referred to in Section 1 and we should be careful to treat it as such.

### 3. Preparation and measurement

As hinted in the previous sections, the point of view taken here is that an experiment is defined by the experimental instruments. Whatever these instruments might be, it will be assumed that they include a clock, a switch, or other mechanism that the experimenter activates in order to start a run, and some recording devices that, for brevity, we shall call dials, whose readings give the result of the experiment.

Very often in practice the instruments have adjustments that are set before the start of a run. Runs with different settings of the adjustments belong to different experiments. One can regard the corresponding experimental setups as being different but it is often convenient not to do so and speak instead of experiments that share the same experimental setup with different settings of the adjustments.

To analyze an experimental run we shall regard it as divided into two phases. In the first phase, called the *preparation*, the experimental instruments are put in place, adjusted if necessary, and the run is started. The second phase, called the *measurement*, follows the first and includes the recording of the appropriate readings of the instrument dials.

In many experiments the instruments that are activated at the start may have been constructed as distinct from those whose dials are read during the measurement phase. But in other experiments this may not be the case. In general, therefore, it is difficult or even impossible to define the two phases by referring to the instruments used in them. Instead the two phases can be defined by selecting an appropriate *reference time*  $\tau$ , read by the laboratory clock, to separate them. The preparation phase is the part of the experimental run that precedes  $\tau$  and the measurement time the one that follows it.

The reference time  $\tau$  is restricted to an interval but otherwise arbitrary. Clearly  $\tau$  must be a time not before the starting switch is closed and not after the dials are read. Although the precise limits of the interval may be in principle very difficult to define, this should not concern us here. In practice, a *preparation time*  $t_0$  and a *measurement time*  $t$  can be adequately, though to some extent arbitrarily, defined by an experimenter with full knowledge of the operation of the experimental instruments. The reference time  $\tau$  may then be taken to lie in the interval  $(t_0, t)$ . When thinking of the collection of runs that constitute an experiment as starting simultaneously, the time  $t_0$  will be the same in all runs but the time  $t$ , not under the experimenter's control, may be different in different runs. We can select the same reference time for all runs by choosing it sufficiently close to  $t_0$ .

To the two phases, preparation and measurement, in which we imagine the experiment divided, we make correspond two apparatus.

The *preparation apparatus* is defined by the experimental setup and the interval  $(t_0, \tau)$ . Its *configuration* is given by the setting of the adjustments.

The *measurement apparatus* is defined by the experimental setup and the interval  $(\tau, t)$ . Its *configuration* is given by the readings of the measurement dials.

One should be careful not to interpret the word “apparatus” as used in these definitions as merely synonymous of “instrument” or “collection of instruments”. The word once meant not only the equipment necessary to accomplish an end but also the processes leading to it. As used here, it refers not only to the collection of experimental instruments but also to the processes they cause during the corresponding time interval. In some cases, as we shall see in the example that follows, it is possible to define each apparatus with reference to distinct experimental instruments, which shall be referred to as preparing or measuring instruments, but this need not be the case in general.

With the conventions just introduced we are now in a position to regard every experimental setup of interest to us as consisting of a preparation and a measurement apparatus. The instruments that enter into their definition, which we assume include a clock, can be constructed from instructions expressed in plain language, and their configurations, as well as the reference time  $\tau$ , can *a fortiori* be described in such a language. The *configuration* of an apparatus will be taken as given by a finite set of numbers (which we can think of as the setting of the instrumental adjustments or the reading of the dials). The theoretical description of a given experiment will depend on the configuration of the apparatus as well as on the reference time chosen. Theoretical descriptions corresponding to different choices of the reference time must, of course, yield identical experimental predictions.

In the body of this book we shall make extensive use of the framework just explained to analyze idealized experiments chosen in such a way as to explain the physical concepts clearly. At this stage however, at the risk of being somewhat less clear, it is preferable to use a realistic example to illustrate how it can be cast into the above framework, for the theory is meant to explain actual experiments.

The example that we shall consider is the “molecular diffraction” experiment schematized in Figure 1.1. The experimental instruments consist of a clock, a supply of helium, a chamber that is equipped with a sensitive pressure gauge, a collimating tube and a shutter, two parallel disks each with a slit cut near its circumference and mounted on a common shaft with their slits offset with respect to each other, a motor that can rotate the shaft with adjustable angular velocity, a crystal of lithium fluoride, another chamber with a hole which is also equipped with a sensitive pressure gauge, and a circular track, on which the second chamber slides, graduated in degrees and centred at the crystal. The first chamber when

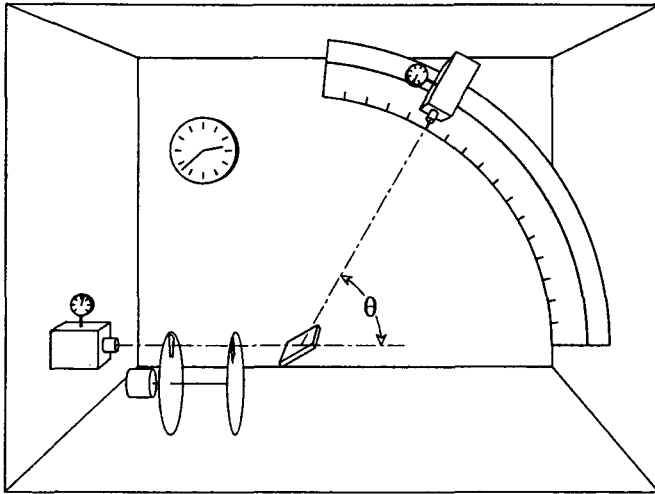


Figure 1.1 Molecular beam diffraction experiment.

filled with helium will be called “the source”, the second chamber, “the detector”. The disposition of the instruments is as shown in the figure.

To start the experiment, the experimenter sets up the source with helium gas at the desired pressure, adjusts the angle  $\theta$  that determines the position of the detector, starts the motor, adjusts the angular velocity of the disks and opens the shutter of the source. A fixed interval of time later the pressure in the detector is noted and the run completed.

One possible choice of reference time would be at a moment just before the pressure in the detector starts to rise. With this choice, the source, rotating disks, crystal and the second chamber are sufficient to define the preparation apparatus. Its configuration is given by the gas pressure in the source, the angular velocity of the disks and the angle  $\theta$ . The only instrument needed to define the measurement apparatus is then the pressure gauge in the detector.

Another choice would be to select the reference time  $\tau$  by doing some preliminary experiments in which a detector is placed behind the slitted rotating disks and the interval  $\delta$  between the opening of the shutter and the activation of the detector is noted. The time  $\tau$  can then be taken to be a time  $\delta$  after the opening of the shutter. With this choice the crystal and the second chamber would be required to define the

measurement apparatus, but note that the angle  $\theta$  that determines the position of the chamber will still form part of the configuration of the preparation apparatus.

In the preceding discussion we have implicitly made a distinction between the experimental setup and the particular settings of the instruments during the preparation phase. Strictly speaking such distinction does not exist. There is no conceptual difference between, say, adjusting the temperature of the oven of our molecular diffraction experiment and replacing it by a different helium filled oven, or by any other device that produces a beam of helium atoms. It is in the extended sense illustrated by this example that we should understand the term "configuration" in reference to the preparation apparatus.

The preceding remarks cannot be extended to the measurement apparatus. When we talk about different configurations of the latter we mean different readings of the measurement dials in a given experimental setup. Experience shows that these readings may be different in experimental runs performed with identical configurations of the preparation apparatus.

Quantum mechanics explains the results of an experiment in the sense that it correctly predicts the correlations between the configurations of a preparation and of a measurement apparatus defined for arbitrary values of the reference time  $\tau$ . In Chapters II and III the mathematical theory necessary for the description of the correlations between the configurations of both apparatus for a given  $\tau$  will be presented, in a sense these theory corresponds to the theory of classical kinematics. In Chapter IV the theory that gives the connection between the descriptions of an experiment for *different* values of  $\tau$  will be introduced. It corresponds to the theory of classical dynamics.

Physics does not only deal with experiments of the type described above. Very often, notably in such fields as astrophysics or meteorology, a preparation apparatus is not constructed. Rather, measurements are performed at different times on, say, the stars or the atmosphere. One can cast this type of observations into the structure that we have described by imagining nature itself to provide a preparation apparatus whose configuration is given by the readings resulting from a group of early observations while the rest of the observations constitute the readings of the measurement apparatus.

There are also experiments in which each run consists of a preparation and several successive sets of measurements. They can be cast into our framework in several ways. Often the most convenient one is to consider the experiment as a series of partial experiments in which each intervening measurement is regarded as a measurement for the previous partial experiment and as a preparation for the next one.