INTRODUCTION

Lotfi A. Zadeh was born in 1921 in Baku, the former Soviet Azerbaijan, where he lived only until 1931. The next 13 years, he lived in Tehran, Iran. He studied at Aborz College (an American Presbyterian Missionary School) and, later, at the University of Tehran, where he received in 1942 a B.S. degree in electrical engineering. In 1944, he came to the United States for graduate studies at MIT, where he received the S.M. degree in electrical engineering in 1946. The same year, he joined Columbia University as an instructor in electrical engineering and was also admitted to the doctoral program. He received his Ph.D. degree in 1949 and served on the Electrical Engineering Faculty of Columbia University until 1959.

During his 13 years at Columbia University, Zadeh made several important contributions to electrical engineering, including a new direction in frequency analysis of time-varying networks, an extension of Wiener's theory of prediction, and a new way of analyzing sampled-data systems, which led to the well-known and widely used z-transformation. He taught courses in circuit analysis, system theory, electromagnetic theory, finite-state machines, and information theory. During this period, he also became interested in the emerging computer technology, whose significance he quickly recognized.

In 1959, Zadeh left Columbia University and joined the Electrical Engineering Department of the University of California at Berkeley. During his early years at Berkeley, he worked on various problems emerging from system theory, including problems of optimal control, time-varying systems, and system identification. Some of this work resulted in a monograph *Linear System Theory: The State Space Approach* (McGraw-Hill, 1963), which he co-authored with Charles Desoer. His papers during this period, as well as his earlier papers written at Columbia University, are listed in Part B of our Bibliography.

In 1963, Zadeh was appointed Chairman of the Department of Electrical Engineering and held the position for five years. In this position, he emphasized the growing importance of computer technology and managed to transform the department into the Department of Electrical Engineering and Computer Science. Since 1965, when he introduced the concept of a fuzzy set, Zadeh's research has almost exclusively been oriented to the development of fuzzy set theory and related areas. His work in these areas, which is the subject of this book, is characterized in more detail later in this biographical overview.

During his academic career, Zadeh also held visiting positions at other institutions, including the Princeton Advanced Study Institute (1956), Electrical Engineering Department of MIT (1962 and 1968), IBM Research Laboratory in San Jose (1968, 1973, and 1977), Artificial Intelligence Laboratory of the Stanford Research Institute at Menlo Park, California (1981), and the Center for the Study of Language and Information of Stanford University (1988). After 1991, when he became Professor Emeritus, he has continued to be active in teaching and research as the Director of the Berkeley Initiative in Soft Computing.

For his enormous contributions to science and engineering, Zadeh has received numerous awards and honors. In 1995, he received the IEEE Medal of Honor, the highest honor given by IEEE, for his "pioneering development of fuzzy logic and its many diverse applications." Among his other awards are the Honda Prize (1989), the IEEE Education Medal (1958), the IEEE Centennial Medal (1984), the IEEE Richard W. Hamming Medal (1992), the ASME Rudolf Olderburger Medal (1993), the Kampe de Feriet Medal (1993), the Grigore Moisil Prize (1993), and several honorary doctoral degrees. He is also a member of the National Academy of Engineering, a foreign member of the Russian Academy of Natural Sciences, and a Fellow of IEEE, AAAS, ACM, and AAAI.

In this book, we are interested only in Zadeh's contributions to fuzzy set theory and related areas. To facilitate our examination of these contributions and to introduce his papers included in this book, we utilize Part A of our Bibliography. This part of the Bibliography contains all papers by Zadeh published in the period 1965–95 that deal with fuzzy set theory and related areas; Zadeh's other papers are listed in Part B of the Bibliography. For convenience of the reader, reference numbers of papers that are included in the two volumes of Collected Papers by Lotfi A. Zadeh (see Preface) are printed in bold, and those included in this volume are identified by the asterisks.

As is well known, fuzzy set theory began its existence in 1965, when Lotfi Zadeh introduced the concept of a fuzzy set in his seminal paper [1]. However, Zadeh is not only the founder of fuzzy set theory, but he has also been one of the most important contributors to the theory during its thirty-year existence. Indeed, he has originated most of the key ideas associated with the theory and conceived of many of its applications. He often developed his new ideas only to some degree, and let others to develop them further.

It is significant, but less known, that Zadeh recognized the need for fuzzy mathematics a few years before he published the seminal paper on fuzzy sets. This recognition, which emerged from his work on system theory, is expressed, for example, in the following quote from his 1962 paper "From Circuit Theory to System Theory" [56] (in Part B of Bibliography):

> ... there is a fairly wide gap between what might be regarded as "animate" system theorists and "inanimate" system theorists at the present time, and it is not at all certain that this gap will be narrowed, much less closed, in the near future. There are some who feel this gap reflects the fundamental inadequacy of the conventional mathematics—the mathematics of precisely-defined points, functions, sets, probability measures, etc.—for coping with the analysis of biological system, and that to deal effectively with such systems, which are generally orders of magnitude more complex than man-made system, we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not described in terms of probability distributions. Indeed, the need for such mathematics is becoming increasingly apparent even in the realm of inanimate systems, for in most practical cases the *a priori* data as well as the criteria by which the performance of a man-made system is judged are far from being precisely specified or having accurately known probability distributions.

In the rest of this overview, we intend to trace the development of Zadeh's ideas pertaining to fuzzy sets, fuzzy logic, and fuzzy systems via his papers. To capture this development, we decided to present the papers in this book in the order in which they were published.

After careful reading of the seminal paper [1], in which Zadeh introduced the notion of fuzzy sets, one can easily recognize that the paper is rich source of ideas that played a fundamental role in the evolution of fuzzy set theory. Some of these ideas were further developed by Zadeh himself in his other papers, some were later developed by others. Let us examine these ideas.

First, it is quite significant that fuzzy complementation, intersection, and union are defined in the paper by operations that are usually referred to as *standard fuzzy set operations*. As is now well known, these specific operations possess several significant properties. Among all possible fuzzy complements, for example, the standard fuzzy complement is the only linear one. Among all possible fuzzy intersections (t-norms) and fuzzy unions (t-conorms), the standard operations are the only operations that are *idempotent*, and also the only operations that are *cutworthy* (i.e., they are preserved in all α-cuts of the fuzzy sets involved). Moreover, the standard fuzzy intersection is the largest fuzzy intersection, while the standard fuzzy union is the smallest fuzzy union. Although Zadeh was apparently aware that fuzzy counterparts of classical set operations are not unique, and mentioned examples of other possible operations in the paper, his remarkable insight allowed him to recognize the significance of the standard operations. He also recognized the existence of aggregation operations whose outcomes are fuzzy sets that contain the standard fuzzy intersection and are contained in the standard fuzzy union. Operations of this kind, which have no counterparts in classical set theory, are now referred to as *averaging operations*. It is significant that the standard fuzzy intersection and standard fuzzy union may also be viewed as the smallest and greatest averaging operations, respectively.

Second, the paper introduces the concept of a *fuzzy relation* and the notion of a *composition* of two binary fuzzy relations that are compatible. For fuzzy relations defined on the n-dimensional Euclidean spaces ($n \geq 2$), it also introduces the concept of *shadows* (or *projections*) on various hyperplanes. The subject of fuzzy relations was further developed by Zadeh in his later papers, especially papers [4] and [12]. In [4], he elaborates on properties of shadows of fuzzy relations on n-dimensional Euclidean spaces. In [12], he introduces fuzzy counterparts of the classical properties of *reflexivity*, *symmetry*, *transitivity*, and *antisymmetry* of binary relations. Using these fuzzy properties, he examines various types of fuzzy relations, such as *fuzzy equivalence*, *compatibility*, and *ordering relations*. All the defined properties and the various types of fuzzy relations based on them are cutworthy.

Third, the paper contains the first formulation of the *extension principle*, even though Zadeh did not use the term "extension principle" in the paper. The principle is introduced in the paper under the heading "fuzzy sets induced by mappings." The term "extension principle" was introduced by Zadeh in [28], where the principle and its utility are thoroughly explained.

Fourth, the paper introduces the notion of *convexity* of fuzzy sets and examines some of its properties. It also introduces the notion of boundedness of fuzzy sets. Both of these notions, which are defined in the paper as cutworthy concepts, were later instrumental in formulating the concept of a fuzzy number and fuzzy arithmetic.

Fifth, although the paper deals with membership functions whose range is the unit interval [0,1], it was recognized by Zadeh that "the range of the membership function can be taken to be a suitable partially ordered set P." He thus anticipated the existence of fuzzy sets that are now referred to as *L-fuzzy sets*.

Sixth, the concept of an α-cut is introduced in the paper and, as previously mentioned, it is utilized for defining convexity and boundedness of fuzzy sets. The *α-cut representation* of fuzzy sets is not explicitly included in the paper; it was formulated by Zadeh a few years later in [12].

Seventh, an extension of the *separation theorem* for classical convex sets to fuzzy sets is introduced in the paper, and the utility of this extension for dealing with the problem of pattern discrimination is discussed. The use of fuzzy sets to pattern discrimination and data clustering, which is only hinted at in the paper, is further examined in [3] (whose early version appeared as RAND Memorandum RM-4307-PR in October 1964) and, more extensively, in [34].

In addition to the seminal paper, which we just discussed, Zadeh also published in 1965 another paper [2], less known but also significant. In this paper, he introduces the concept of a *fuzzy system* and discusses the problem of optimizing crisp systems under fuzzy constraints. Zadeh's interest in fuzzy systems has remained strong, as documented by several other papers he has published on the subject [10, 21, 22, 24, 49, 100]. He also introduced various new ideas regarding fuzzy sets in these papers. In [10], for example, he introduced two important ideas, the idea of a fuzzy graph and the idea of representing fuzzy sets by n-dimensional unit hypercubes. The notion of *fuzzy control* was introduced for the first time in papers [15] and [20]. The concept of a *fuzzy finite-state machine* was introduced in [10].

In one of his early papers on fuzzy sets [5], Zadeh introduced the concept of a *fuzzy algorithm* and the associated concept of a *fuzzy Turing machine*. In another paper [16], he introduced the concept of a *fuzzy Markoff algorithm*. Although he addressed various issues regarding fuzzy algorithms in other papers (e.g., [10, 14, 22, 28]), this subject is still rather underdeveloped.

Another important concept recognized by Zadeh in his early papers [8, 11, 14, 18] is the concept of a *fuzzy language*. Although he did not develop fuzzy languages beyond their coverage in these four papers, they were further developed by other researchers and applied to pattern recognition and other areas.

It is easy to recognize that the important and broad area of *fuzzy decision making* was initiated by a key paper [9], which Lotfi Zadeh co-authored with Richard Bellman. The paper is a rich source of ideas regarding fuzzy decision making, including *fuzzy dynamic programming*. Zadeh presented further ideas concerning fuzzy decision making and *fuzzy optimization* a few year later in another paper [29].

A very important contribution by Zadeh was his introduction of *possibility theory* as a measure-theoretic calculus for dealing with information expressed by fuzzy propositions [36]. He also demonstrated the great utility of possibility theory in several other papers, including [35, 37, 44, 46, 48, 50, 52, 53]. In [37], he formulates a meaning representational language PRUF (an acronym for Possibilistic Relational Universal Fuzzy) for natural languages, which is based on possibility theory. In [52, 65], he shows how PRUF can be employed for determining the meaning of linguistic terms of natural language by possibility distributions.

Another important idea due to Zadeh is the concept of a *linguistic variable*, introduced initially in [22]. This paper is also a source of many other basic ideas that underlie most of the current applications of fuzzy logic. Among these are the calculus of fuzzy if-then rules and fuzzy algorithmic description of dependencies and commands. This paper and the 1965 paper on fuzzy sets [1] have been cited by the Citation Index as "Citation Classics." In addition, paper [22] has been identified by the Citation Index as the most frequently cited paper among all papers that have been published in the *IEEE Transactions on Systems, Man and Cybernetics.*

The concept of linguistic variables was fully developed by Zadeh later, in a significant, three-part paper [28]. Virtually all material covered in this extensive paper is directly applicable to *fuzzy logic* in its broad sense, as a system of concepts, principles, and methods for dealing with modes of reasoning that are approximate rather than exact. The paper also introduces the intriguing concept of a *fuzzy theorem*, a potentially very important concept, which still remains totally undeveloped.

Fuzzy logic in its broad sense was addressed by Zadeh in a paper presented at the 1974 IFIP Congress [25]. It is basically viewed as an application of tools developed within fuzzy set theory to approximate reasoning. These tools include aggregation operations on fuzzy sets, fuzzy numbers and fuzzy arithmetic, fuzzy relations and fuzzy relation equations, α-cut representations of fuzzy sets, the extension principle, and the like.

The development of basic ideas (principles, methods, etc.) of *approximate reasoning*, which has occupied Zadeh since the mid 1970s, is another of his principal contributions. In addition to the mentioned paper [28], in which a connection is established between approximate reasoning and linguistic variables, he wrote several broad papers on approximate reasoning focusing on different issues [26, 27, 30, 43, 64, 72, 89, 98]. He also explored the role of approximate reasoning in *expert systems* [58, 79, 95]. Some of his papers deal with special aspects of approximate reasoning, such as *reasoning with dispositions* [60, 66, 69, 76, 84, 86, 88], the concepts of *usuality* [74, 76, 80, 85], *information granularity* [40], *fuzzy quantifiers* [28, 55], or *the calculus of fuzzy if-then rules* [98]. More recently, Zadeh proposed the notion of *soft computing* [99, 102], whose aim is to exploit the tolerance for uncertainty to achieve tractability, robustness, and low cost. In soft computing, fuzzy logic is employed in conjunction with other tools, such as neural networks, genetic algorithms, and theories of imprecise probabilities.

In several papers published in the late 1970s and early 1980s, Zadeh addressed the issue of determining the right representation of linguistic terms (in given contexts) by fuzzy sets or, in other words, the issue of constructing appropriate membership functions. In addition to the three already mentioned papers that describe the use of the representational language PRUF for this purpose [37, 52, 65], at least the following papers belong to this category [32, 54, 57, 59, 63, 79, 81].

One additional subject initiated by Zadeh should be mentioned, the joint use of fuzziness and probability. In one of his early papers [6], he introduced the concept of *probability measures of fuzzy events*. Later, he introduced the concept of *fuzzy probabilities* [51, 70]. He also showed, in his various papers on approximate reasoning, how these concepts can be utilized for dealing with probability-qualified fuzzy propositions. He also published debate papers [47, 62, 78], in which he attempts to clarify the various misconceptions regarding the relationship between fuzzy set theory and probability theory.

At the time of publication of this book, Zadeh's primary interest is in developing a methodology in which words are used in place of numbers for computing and reasoning. This subject, which he refers to as "computing with words," is a further extension of approximate reasoning.

Although our overview of Lotfi Zadeh's work on fuzzy set theory and related subjects is not fully comprehensive, we hope that it will help the reader to get sufficiently oriented to comprehend the enormous contribution to this field by one of the most outstanding scholars of this century.

Lotfi Zadeh during his student years in Tehran in the early 1940s (the large Russian sign ODIN, which means "alone," was his early proclamation of independence).