PREFACE

Many years ago I began to write a book about special relativity and its relation to classical mechanics and electromagnetism. The first draft got longer and longer and when I persuaded my colleague Dr.M.G.Bowler to read it, he thought that it was so long that he could not imagine who might want to read it. I rather agreed with him and set the whole thing aside. Years later, as an examiner, I found that although most students could manipulate the formulae of special relativity, very few of them could explain, even qualitatively, where the notion of a variable inertial mass $m_o/\sqrt{(1-v^2/c^2)}$ came from. It then seemed to me that there might be room for an elementary book on special relativity that emphasised the basic physical ideas, their consequences and their applications other than in particle physics, where there are already two excellent texts by Hagedorn and Muirhead (see bibliography).

My plan is to give a straightforward account, using only elementary mathematics, of the basis of special relativity, and then to stress its important consequences and those aspects of relativity relevant to physics and science in general.

I share Muirhead's view that most books on relativity over-emphasise the "rods and clocks" aspects of the subject. I have a dim memory of one book that asked me to visualise the whole of space being strewn with clocks much, I suppose, as a sheet of graph paper is strewn with millimetre squares! This emphasis probably derives from the time, many years ago, when there were no reproducible atomic clocks and a great deal of circumlocution was needed to explain how identical but independent clocks and measuring rods could exist in two systems in rapid relative motion. The result is usually to create the impression that understanding relativity means abandoning <u>all</u> common-sense notions about space and time. I hope to show that this is false, and that the entire theory of special relativity can easily be developed from a few simple ideas even if, eventually, it has some rather bizarre consequences.

The whole aim of special relativity is to reconcile the notions implicit in Newton's first law of motion (the basis of classical dynamics) with the universal invariance of the velocity of light in vacuum (a direct consequence of Maxwell's electromagnetism). It does this by changing how time intervals and spatial distances between two events are related when they are given in two frames of reference in relative motion. This is all encapsulated in a simple formula, the Lorentz transformation, and this formula is the keystone of special relativity. The first two chapters explain how this comes about.

In deriving the Lorentz transformation I first explain why it has to be linear, something that seems to be ignored by every author except Fock (1959). Thereafter I arrange the steps in the derivation to show how much of it depends only on assuming that there are no preferred positions or directions in space, and that all instants in time are equivalent. This makes it possible to see how one further assumption, that there is a universal limiting velocity *c*, leads to the Lorentz rather than the classical Galilean transformation. To agree with Maxwell's equations and experiment *c* must, of course, be the velocity of light.

Chapter 3 describes several of the optical and kinematic effects of relativity. Some, such as time dilation, seem strange when we try to reconcile them with those naive ideas about time and space that arise from our experience of slow motion, slow, that is, compared with the velocity of light which is about a billion (10^9) km/hr. These effects are, nevertheless, confirmed by numerous experiments.

The most important consequence of special relativity is, however, not these kinematic effects but its effect on dynamics. It alters the conservation laws for momentum, mass and energy and the response of particles to forces.

Chapter 4 reviews classical dynamics in preparation for relativistic dynamics in Chapters 5 and 6. By the end of Chapter 6 the reader will have met most of the simple kinematic and dynamical effects of relativity, and thus understand the origin of the relation $\mathbf{p} = \gamma m_o \mathbf{v}$, and the significance of the famous formula $E=mc^2$.

Up to this point only the simplest mathematics has been used but from now on slightly greater demands begin to be made on the reader's mathematical knowledge. This might therefore be a good place to end a first course in special relativity.

Further important consequences of relativity that should form part of a physicist's general knowledge are the new insights that it provides into the structure of electromagnetism, and the way it modifies the principle of least action, the branch of classical mechanics leading most directly to quantum mechanics.

At this stage it is becoming clear that we need a better, more compact notation and so 4-vectors are introduced in Chapter 7. Since students find it easier to follow, we adopt the Minkowski notation in which an event is located at x_1, x_2, x_3, x_4 with x_4=ict, rather than introduce covariant and contravariant components and a metric tensor. The Minkowski notation would present problems if we intended to pursue relativistic wave mechanics, where i, the square root of -1, has already been pre-empted for another purpose. The use of 4-vectors allows us to write equations in a form that makes their agreement with the principle of relativity immediately obvious. Since both angular momentum and electromagnetism involve the vector product, we also need 4-tensors.

The use of 4-dimensional notation has been postponed to this late stage, to emphasise that it is merely a book-keeping aid, and a convenience. It has no more to do with the physics of relativity than the use of complex numbers has to do with the physics of A.C. circuits. A good deal of loose talk about a 4-dimensional world is caused by assuming that its physics is determined by notation.

Electromagnetism is treated in Chapters 8 and 9. Since we already know that relativity is consistent with classical electromagnetism, it can lead to nothing fundamentally new, although its effect on dynamics will alter the motion of charged particles in response to fields. Relativity can, however, lead to new and more concise ways of deriving classical results, for example, the treatment of synchrotron radiation in #9.8 is very much simpler than the classical treatment using the Liénard-Wiechert potentials.

Chapter 10 is partly about how the energy-momentum tensor combines fields, particles and continuous media in a single Lorentz invariant formulation yielding several general results, such as the centre of mass theorem. This chapter also discusses how relativity affects angular momentum, and its separation into orbital and spin components.

Chapter 11, on the principle of least action and the Hamiltonian ends with a sketch of how combining special relativity with quantum mechanics leads to electron spin and thus to the periodic table of the elements, to chemistry and to biology. As our existence depends on the chemistry of the elements, we may regard this as a particularly important consequence of relativity.

A few mainly mathematical topics, and also some aspects of the linearity of the Lorentz transformation, have been relegated to an appendix. The appendix ends with a brief section, of a mainly epistemological nature, and a table of physical constants.

There are problems at the end of each chapter and answers towards the end of the book.

References to authors quoted in the text are collected in alphabetical order at the end of the book, together with a short bibliography.