

work hardening, etc. etc. It provides not only a deeper qualitative physical picture of plastic deformation but also to a certain degree a quantitative analysis of it.

Plastic deformation can also occur by twinning. The atoms slide, layer by layer to bring each deformed slab into mirror-image lattice orientation relative to the undeformed material. The critical stress of twinning is usually higher. Twins form at low temperature and under rapid deformation, e.g. bcc iron strained quickly at room temperature and slowly at 100K.

### 1.2. Griffith Criterion: Role of Surfaces

Griffith (1924) derived an expression for the elastic crack propagation on the basis of thermodynamic considerations. He reasoned that a crack would advance when the incremental release of stored elastic strain energy  $dW_E$  in a body became greater than the incremental increase of surface energy  $dW_s$  as new crack surface was created. For the two-dimensional case in plane stress

$$W_E = \frac{\pi\sigma^2 c^2}{E} \quad (1.2.1)$$

$$W_s = 4c\gamma_s$$

where,  $\sigma$  is the nominal stress;  $E$ , the elastic modulus;  $2c$ , the length of the crack, and  $\gamma_s$  the specific surface energy.

The Griffith criterion can then be written as with  $\sigma_F$ , the fracture stress,

$$\sigma_F = \sqrt{\frac{2E\gamma_s}{\pi c}} \quad (1.2.2)$$

by the condition that

$$\frac{\partial}{\partial c} W_E \geq \frac{\partial W_s}{\partial c}.$$

Subsequent analysis\* in fracture mechanics defines a parameter, crack extension force,  $G = K^2/E$  (in plane strain) being equal to a critical value,  $G_{Ic}$ ,

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\*This is associated with the names of Irwin (1957) and Inglis (1913). The analysis given by Inglis has been generalized by R. LÖfstedt (Phys. Rev. *E55*, 6726, 1997) who has proposed an inequality involving a ratio of time scales to determine whether a material is brittle or ductile. One time scale is 'ductile', and is associated with the rate of decrease of the tensile stress at the tip of a narrow crack. The above ductile time scale is to be compared with a characteristic 'phonon time'  $a/v_s$ , where  $v_s$  is the velocity of sound, and  $a$  measures the lattice spacing. See also R. W. Armstrong (Mat. Sci. Eng. *1*, 251, 1996) also A. Kelly, W. R. Tyson and A. H. Cottrell, Phil. Mag. *15*, 567, 1967 and J. R. Rice and R. Thomson, Phil. Mag. *29*, 73, 1974).

the crack-resistance force of the material. For the elastic crack in an infinitely wide plate

$$G_{Ic} = \frac{K_{Ic}^2}{E} = \frac{\sigma_F^2 \pi c}{E} \quad (1.2.3)$$

where  $K$  is the stress intensity factor:  $K_{Ic}$  is called fracture toughness.

Comparing to Eq. (1.2.2),  $G_{Ic} = 2\gamma_s$  — the two approaches lead to the same result although their methods are different. The specific surface energy plays a very important role in brittle fracture.

Engineering materials do not fracture in a completely elastic manner. The localized plastic deformation near crack tip gives the materials some toughness, or resistance to crack propagation.\* Orowan (1948) proposed to add a term  $\gamma_p$ , the plastic work expanded during crack propagation to the elastic work  $\gamma_s$  as an effective specific surface energy in Eq. (1.2.2). The Griffith equation is modified to read (in plane stress)

$$\sigma_F = \sqrt{\frac{2E}{\pi c}(\gamma_s + \gamma_p)} \sim \sqrt{\frac{2E\gamma_p}{\pi c}}. \quad (1.2.4)$$

(Some authors wrote  $2(\gamma_s + \gamma_p)$  as  $2\gamma_s + \gamma'_p$ ; then,  $\gamma'_p = 2\gamma_p$ .) From Eq. (1.2.4), it seems  $\gamma_s$  is no longer an important factor in this process. However, Tetelman *et al.* (1967) showed that for the case of Fe-3% Si, by Frank-Read source multiplication (Cottrell 1964a):

$$\gamma_m = \text{const.} \cdot \gamma_s N_0^{\frac{3}{2}} \left(\frac{v_s}{v_c}\right)^2 T^{\frac{5}{2}} \quad (1.2.5)$$

where  $\gamma_m$  is defined as the product of the work done in a unit volume element of material when the crack advances and the distance perpendicular to the crack in which the deformation is extensive.  $N_0$  is the density of mobile dislocation sources,  $v_s$  and  $v_c$  are velocities of sound and the crack respectively.  $\gamma_m$ , like  $\gamma_p$  is a measure of the intrinsic toughness of a solid. In Eq. (1.2.5),  $\gamma_s$  is a multiplying factor and not an addition term. The change of  $\gamma_s$  directly influences the change of  $\gamma_m$ .

Moreover, Lung and Gao (1985) calculated the relative  $K_{ic}$  value of metals with a simplified dislocation motion model and BCS dislocation distribution function at the crack tip

$$G_{ic}^p \cong 2\gamma_p \propto W_i = E_0 (K_{ic}^0)^2 F_i(\theta_0) r_i^*(\theta_0)^{\frac{1}{2}} \quad (1.2.6)$$

\*A general article on failure of solids is that of M. Marder and J. Fineberg (Phys. Today, September 1996, p. 24).

where  $K_{ic}^0$  is the fracture toughness in linear elastic case ( $= \sqrt{EG_{ic}^0}$  or  $\sqrt{2\gamma_s E}$ );  $r_i^*$ , the plastic zone size; and  $E_0 \propto E^{-1}$ . The  $E_0$  in Eq. (1.2.6) is proportional to the inverse of the elastic modulus of materials, and  $F_i(\theta)$ , the angular dependent function respectively.  $\theta_0$  is the direction of  $r_{\max}^*$  of the plastic zone.

Comparing Eqs. (1.2.6) with (1.2.5), the two approaches lead to the same conclusion that  $\gamma_s$  plays the role of a multiplying factor in the expression of critical crack extension forces. For a multiplying factor,

$$\frac{\Delta(\gamma_s f)}{(\gamma_s f)} = \frac{\Delta\gamma_s}{\gamma_s} + \frac{\Delta f}{f}. \quad (1.2.6)'$$

The relative change of  $\gamma_s$  is as important as that of  $f$ . If we consider the underlying role of atomic forces in the structure of dislocation core and dynamics, the role of interatomic forces is not only in the surface energy term but also in the dislocation core structure.

### 1.3. Peierls Stress and Barrier

A dislocation experiences an oscillating potential energy as it glides in a crystal. In the Peierls model (Peierls, 1940), the bonds across the glide plane were considered to interact via an interatomic potential, while the remainder of the lattice was linearly elastic. Nabarro (1957) gave an analytical expression for the dislocation core model. One can approximately estimate the ideal lattice resistance to dislocation motion by means of the Peierls model. The resolved applied stress necessary to move the dislocation over the Peierls barrier is called the Peierls stress,  $\sigma_p$ . The Peierls stress comes from the expression for the Peierls energy which changes for a translation of the dislocation by a distance smaller than the Burgers vector.

Figure 1.1, reproduced from Nabarro (1967), shows the Peierls model of a dislocation. The material above A and below B is regarded as forming an elastic continuum. The force between the rows A and B is a periodic function of the displacement.

As the dislocation moves through the lattice, it passes through an unsymmetrical configuration to a different symmetrical configuration in which one half plane of atoms on the expanded side of the glide plane lies midway between two half planes on the compressed side. Further motion passes through unsymmetrical configurations back to a state equivalent to the original. The dislocation moves if a finite force acts on it. The critical stress is the Peierls stress. After a lengthy calculation, the approximate energy of misfit