

*Dedicated to my teacher
Yurii I. Manin*

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PREFACE

This book is an introduction to the soliton theory from a mathematical point of view. It is a study of general integrable two-dimensional systems of differential equations and related (mostly algebraic) structures. The very first papers on the integration of the celebrated Korteweg - de Vries equation demonstrated the algebraic character of the remarkable new phenomenon. Later the algebraic (in a broad sense) methods of soliton theory became a relatively independent direction close to differential algebra, infinite dimensional representation theory, and the theory of algebraic curves. The book contains a systematic theory of local conservation laws, Bäcklund transformations, algebraic-geometric solutions, and the inverse scattering technique. We constantly use the zero curvature representations and the necessary elements of the theory of loop groups, though the latter are not discussed here. They will play the main role in the next book on the so-called adèle approach in the soliton theory. On the other hand, we always try to adapt general methods utilizing special symmetries of concrete equations as much as we can. *All non-integrable equations are non-integrable the same way, all integrable ones are integrable in their own way.*

The book is oriented towards mathematicians and specialists in "one-dimensional" mathematical physics. The reader is not required to be acquainted with nonlinear equations, solitons, chiral fields. We use only basic mathematical knowledge of undergraduate/graduate calculus and algebra. Each section is rather autonomous to make it possible to study a particular question without reading the whole book. The same holds true (with some reservations) for the concrete equations (the equation of chiral fields, the Heisenberg magnet, Sin-Gordon, the nonlinear Schrödinger equation and others).

One can use the book to master the modern soliton technique. However it is not quite a text book. We mostly tried to present a compact updated introduction interesting to the specialists. We hope that it can draw attention to open questions and stimulate research in the soliton theory.

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