Preface

Few cross interactions in science have been as successful as that of physics and the mathematical branches of differential geometry, algebraic and low dimensional topology. This book is a contribution to the ongoing dialogue between these sciences, through the research of the author over the last seven years. The interaction between these subjects has been a dominant feature of research developments in the past few years, so much so in fact, that no single book can cover them all. I have chosen to present material on two specific instances in which such developments have occurred in the last decade or so and for which I have been, to some extent, involved. The material thus covered in this book centers on Chern-Simons-Witten theories, also known as topological quantum field theories, or, simply, as quantum topology. This topic constitutes the first part of the book, comprising eight chapters. They are written in a purely mathematical style and are intented to serve as an introduction to our physicist readership; we take the reader on a rapid trip through the elements of manifold theory, differential geometry, particularly the fundamentals of Lie groups, fibre bundles and connections, and algebraic topology with emphasis on mapping class groups; and homotopy and homology groups with emphasis on de Rham cohomology. Much of this material is brough together in chapters five, six, and seven.

Chapter one provides an introduction to three-manifold invariants with detailed analysis of the Chern-Simons invariant, the η-invariant. Mapping class groups are put to use in the description of three-manifolds invariant in chapter two using a construction by Khono which largely invokes the techniques of Moore and Segal on conformal field theories. Chapter three was written with the goal of providing introductory material to physicists on the relationship between Teichmüller space, moduli space, and mapping class groups. The interplay between them is discussed at great length, along with the algebraic and homological characteristics they share. This material is extended to chapter four, where we investigate the cohomology of mapping class groups, and their p-torsion. We also study the question of how stable can mapping class groups be, using Harer's homological stability theorem. The full fledged role of mapping class groups in physics is discussed, of course, in chapter two, were we find a three-manifold invariant derived from it, but furthermore, in the second half of the book, namely in chapter 13 entitled Mapping Class Groups and Global Anomalies. The first part of this book ends with a chapter on the geometric quantization of Chern-Simons-Witten theories.

The second half of the book essentially deals with anomalies. We begin with an independent chapter on the relationship between deformation quantization and the occurence of global anomalies. The book then unfolds in an extensive study of chiral and gravitational anomalies, and anomalies and the index theorem, respectively in chapters 10 and 11. Global anomalies of both gauge and gravitational nature are introduced in chapter 12. Here, we investigate Witten's celebrated SU(2) global gauge anomaly, but also the manifestation and cancellation of global gravitational anomalies in some ten and six dimensional supergravity theories. A case study of global gravitational anomalies is made in the context of the ten dimensional heterotic string theory. Chapter 13 provides a detailed analysis of the manifestation of global anomalies in Chern-Simons-Witten theories in relation to three dimensional mapping class groups. The analysis of global gravitational anomalies in ten dimensional physics requires the existence of a special class of spheres, the so-called exotic spheres. Exotic spheres, namely the Milnor-Kervaire ones are explicitly used in Witten's formula for the anomaly cancellation as exhibited in chapters 12. Louis H. Kauffman has generously contributed, in its entirety, a chapter describing the construction of exotic spheres through characteristic classes. The interested reader is invited to read it in chapter 14.

Much is left unsaid in this book about the ongoing interaction between mathematics and physics. I would have loved to write on additional topics in which I have been involved, for instance about very exotic spheres and their role as gravitational instanton and/or soliton and the now available technique to detect them, or the role of classical invariant of knot–such as the Arf invariant– in detecting exotic structures, from dimensions three to eleven. But I realized that writing a monograph seems to invariably result in limiting one to a lesser scope.

The number of people that have contributed to this endeavor is truly astonishing. My gratitude goes to my friend and colleague Lou Kauffman for taking the time to write chapter 14; for reading the original manuscript and for providing critiques which have resulted in an improved manuscript, particularly chapters 8 and 9. I wish to acknowledge the expert proof-reading of Arthur Greenspoon of the American Mathematical Society. Arthur is to be thanked for several readings of the manuscript. The book, *Quantum Topology*, which I co-authored with Kauffman in 1993, benefited a great deal from Arthur's proof-reading work. His contribution is greatly appreciated here. Anne-Marie Piché is also to be thanked for her first draft corrections of early chapters writing. I had help from Zheng Huang, Gelato Sergio, and Orlando Alvarez in setting up the necessary OzTeX version of LaTeX and related software necessary for writing the entire manuscript on my Macintosh Powerbook. Their help is very much appreciated. I have been fortunate enough to find a stimulating and supportive environment at Berkeley. I therefore wish to thank Robert Cahn, my colleagues in the physics and mathematics departments, Irving Kaplansky and the staff of the Mathematical Sciences Research Institute for providing such a thriving environment.

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