

# CONTENTS

<b>PREFACE</b>	<b>vii</b>
<b>1 BASIC NOTIONS AND DEFINITIONS</b>	<b>1</b>
1.1 Signals and Systems . . . . .	1
1.2 Basic properties of solutions . . . . .	9
1.3 Stability notions . . . . .	10
1.3.1 Attracting sets and attractors . . . . .	15
1.4 Classification of attracting limit sets . . . . .	16
1.5 Structural stability and bifurcations . . . . .	16
1.5.1 Basic types of bifurcations . . . . .	17
1.6 Routes to chaos . . . . .	19
1.7 Asymptotic behavior, attractors, limit sets — what can an electronic engineer see in practice? . . . . .	20
1.7.1 How to recognize the behavior . . . . .	22
<b>2 QUANTIFYING DYNAMIC BEHAVIOR</b>	<b>25</b>
2.1 Lyapunov exponents . . . . .	25
2.1.1 Lyapunov exponents for a discrete-time dynamical system . . . . .	28
2.1.2 Lyapunov exponents for a continuous-time dynamical system . . . . .	29
2.1.3 Lyapunov exponents and the type of attractor . . . . .	29
2.2 Attractor dimension . . . . .	29
2.2.1 Topological entropy . . . . .	31
2.2.2 Temporal auto-correlation function . . . . .	32
2.3 Chaos — definition problems . . . . .	35
<b>3 CHAOTIC SIGNAL ANALYSIS AND PROCESSING</b>	<b>39</b>
3.1 Reconstruction of system dynamics from measured time series . . . . .	39
3.1.1 Topological embeddings . . . . .	39
3.1.2 Differentiable embedding . . . . .	40
3.1.3 Examples of reconstruction . . . . .	41

3.2	Observers for chaotic systems . . . . .	42
3.2.1	Linear observers . . . . .	43
3.2.2	Chaos observers . . . . .	46
3.3	Characterization of chaos by unstable periodic orbits . . . . .	46
	How to find unstable periodic orbits using experimental data? . . . . .	47
<b>4</b>	<b>ELECTRONIC CIRCUITS GENERATING CHAOS — A BRIEF OVERVIEW</b>	<b>53</b>
<b>5</b>	<b>EMPIRICAL METHODS FOR STUDYING CHAOTIC SYSTEMS</b>	<b>57</b>
5.1	Laboratory test equipment . . . . .	57
5.1.1	Simplest Laboratory Experiments . . . . .	58
5.1.2	Advanced trajectory observations using an oscilloscope . . . . .	58
5.1.3	Oscilloscope observations in the RC-ladder chaos generator . . . . .	60
5.2	Simulation experiments for observing trajectories . . . . .	65
5.3	Investigating bifurcation phenomena . . . . .	69
<b>6</b>	<b>ANALYTICAL ANALYSIS OF QUALITATIVE BEHAVIOR</b>	<b>81</b>
6.1	Analysis of system geometry . . . . .	82
6.2	Multilevel oscillations . . . . .	88
6.3	Chaos in the Shil'nikov sense . . . . .	91
6.3.1	Homoclinic orbits in the RC-ladder chaos generator . . . . .	95
<b>7</b>	<b>POINT MAPPINGS</b>	<b>99</b>
7.1	Two-dimensional map and its properties . . . . .	100
7.2	Approximate one-dimensional map . . . . .	104
7.2.1	The deformed spiral map . . . . .	110
7.2.2	Properties of the squeezed spiral map . . . . .	112
7.2.3	Chaos creation mechanism . . . . .	132
<b>8</b>	<b>CONJECTURE ON EXISTENCE OF CHAOTIC OSCILLATIONS</b>	<b>141</b>
8.1	Conjecture on chaos generation . . . . .	141
8.1.1	Verification of the conjecture — examples . . . . .	142
	Chua's circuit . . . . .	142
	"Folded torus" circuit . . . . .	149

RC-ladder network with nonlinear feedback . . . . .	151
Single diode chaos generator . . . . .	151
<b>9 COMPLEX BEHAVIOR IN DIGITAL FILTERS</b>	<b>155</b>
9.1 Digital Filtering and Complex Behavior in Digital Systems . . .	155
9.2 Digital filtering . . . . .	156
9.2.1 What is considered as complex behavior in digital systems? . . . . .	157
9.3 Finite Word-length Effects in Digital Filters . . . . .	159
9.3.1 Sources of nonlinearities in digital filters . . . . .	159
9.3.2 Quantization rules . . . . .	161
9.3.3 Overflow schemes . . . . .	161
9.4 Chaotic Behavior in Digital Filter Sections With 2's Complement Arithmetic . . . . .	163
9.4.1 Simulation results — self-similar patterns of trajectories . . . . .	165
9.4.2 Analysis via symbolic dynamics . . . . .	165
9.4.3 Properties of the set $I_\gamma$ . . . . .	169
9.4.4 Analysis using modern mathematical tools . . . . .	169
Measure-theoretic theory . . . . .	169
9.4.5 Higher-order filters . . . . .	171
9.5 Final State Sensitivity in Digital Filters with Saturation Arithmetic . . . . .	178
9.5.1 Parametric analysis of system's behavior: Arnold tongues and devil's staircase . . . . .	178
9.5.2 Circle-map theory and its applications . . . . .	181
9.5.3 Invariant sets and limit sets of trajectories. Analysis of system dynamics via a . . . . . one-dimensional map . . . . .	183
Singular cases . . . . .	185
Invariant sets . . . . .	185
Limit sets of system trajectories . . . . .	186
9.6 Conclusions . . . . .	189
<b>10 SYNCHRONIZATION OF CHAOTIC CIRCUITS     AND APPLICATIONS</b>	<b>191</b>
10.1 Synchronization in chaotic systems . . . . .	191
10.2 Linear coupling . . . . .	192
10.2.1 Uni-directional coupling . . . . .	192
10.2.2 Mutual (bi-directional) coupling . . . . .	194

10.2.3	Synchronization via linear coupling — examples . . .	195
10.2.4	Partial synchronization . . . . .	196
10.2.5	Examples of partial synchronization. . . . .	197
10.2.6	Pecora-Carroll drive-response concept . . . . .	199
	Example — synchronization of Chua's circuits using the drive-response concept . . . . .	201
10.3	Possible applications in communication . . . . .	202
10.3.1	Chaotic switching . . . . .	203
10.3.2	Chaotic masking — secure communication . . . . .	206
10.3.3	Chaotic modulation — spread-spectrum transmission . . . . .	207
	Inverse system approach . . . . .	207
	Example . . . . .	208
10.4	Conclusions . . . . .	209
<b>11</b>	<b>CHAOS CONTROL</b>	<b>211</b>
11.1	Fundamental properties of chaotic systems and goals of the control . . . . .	213
11.2	Simple techniques for suppressing chaotic oscillations — change in the system design. . . . .	214
11.2.1	Effects of large parameter changes. . . . .	214
11.2.2	“Shock absorber” concept — change in system structure. . . . .	216
11.3	External perturbation techniques . . . . .	216
11.3.1	“Entrainment” — Open loop control . . . . .	216
11.3.2	Weak periodic perturbation. . . . .	218
11.3.3	Noise injection. . . . .	218
11.4	Control engineering approaches . . . . .	219
11.4.1	Error — feedback control. . . . .	219
11.4.2	Time-delay feedback control (Pyragas method). . . . .	220
11.5	Control in terms of stabilizing unstable periodic orbits. . . . .	221
11.5.1	Ott-Grebogi-Yorke approach (OGY) . . . . .	221
	Implementation problems for OGY technique . . . . .	227
11.5.2	Sampled input waveform method. . . . .	227
11.5.3	OPF (occasional proportional feedback) — analog chaos controller . . . . .	228
11.6	Improved electronic chaos controller . . . . .	231
11.7	“Chaos-to-chaos” control — Synchronization as a control problem. . . . .	235

11.8 Electronic implementations — are these possible? . . . . .	238
11.8.1 Example — Implementation problems for the OGY method . . . . .	239
Effects of calculation precision . . . . .	240
Approximate procedures for finding periodic orbits . . . . .	242
Effects of time delays . . . . .	243
11.9 Conclusions . . . . .	243
<b>REFERENCES</b>	<b>245</b>
<b>INDEX</b>	<b>273</b>