

INTRODUCTION

*(Theory of nonlinear oscillations. Invariance and isomorphism
of the oscillating processes in systems of varying
physical nature. Nonlinear and parametric phenomena)*

The complex natural phenomena that seemed incompatible in the past have been subject to such interpretation in recent years that they have been „brought down” to the level of our day-to-day research practices and have become part and parcel of them. This is mostly valid for Nonlinear Dynamics. Some new scientific disciplines providing invariant description of effects and phenomena, as regards the material carrier, have emerged – such as Nonlinear radio engineering, Nonlinear Radiophysics, Nonlinear Astrophysics, Nonlinear Optics; Astrodynamics is closely intertwined with Radiophysical wave dynamics, etc. Periodical orbits with a large number of harmonics and the dependence of the period on the amplitude have proved to be regularities characteristic both of planetary systems and of „ordinary” radiophysical oscillating systems. Given a relevant choice of coordinates in systems of varying physical nature (electric, mechanical, acoustic, biological, etc.), processes can be described by the same differential equations and their theoretical oscillating characteristics coincide in form. It is in this sense that we view the isomorphism in systems of varying physical nature. In recent years it has become clear that in the case of typical nonlinear situations – both in Radiophysics and Mechanics and in many other disciplines – it is impossible to foresee the dynamic properties over a period of arbitrary duration even in the conditions of slight perturbations, i.e. a possibility has been discovered of creating chaos in the oscillating processes (peculiar generation of chaotic motions in the absence of random external actions) [3–31]. It has been shown that chaos can appear even in oscillating systems with 1.5 degrees of freedom. This has thrown new light on the issues of steadiness, stability and reliability of the oscillating systems.

At the same time, the opposite research scheme – synergetic self-organization of the systems, has also evolved [32–38]. It has been shown that synergetic grouping of oscillating systems in stable formations is possible even in the case of a limited number of degrees of freedom of these systems.

The fundamental research conducted in the past years in the field of the nonlinear theory of oscillations has resulted in the formation of qualitatively new views as regards the oscillating processes and phenomena. They have evolved to give rise to the Catastrophe Theory and Theory of bifurcations [39–41], the doctrine of self-organization of inorganic systems, the new interpretation of the synchronization phenomenon as one of the mechanisms of self-organization [42], the new ideas of

determined and chaotic oscillations as a manifestation of different facets of the common nature of dynamic systems. The issue of the self-organization of systems has been recognized as one of the most important scientific problems of the last quarter of the twentieth century [43]. The present-day combination of approaches and the unity of the stochastic and synergetic processes create possibilities for establishing, figuratively speaking, an engineering-and-physical laboratory, where various phenomena can be simulated and studied, and real physical analogues involving virtually arbitrary radiophysical and physico-technical processes can be demonstrated. Nowadays all those phenomena are described with one word: complexity.

The Theory of oscillations, as a separate scientific discipline, deals with the investigation of the most general properties and regularities of the processes and phenomena in dynamic systems.

The theoretical and methodological basis of the Theory of oscillations and, above all, of the modern development of the theory and technology of nonlinear and parametric systems includes, parallel with the classical works of Poincaré, Lord Rayleigh and Melde from the end of the 19th century, the classical works from the 1920s, 1930s, and 1950s written by Van-der-Paul (the Netherlands), Barkhausen and Müller (Germany), Epilton (UK), Koga and Hayashi (Japan), Lenard and Karatanou (France), Mandelshtam, Papaleksi, Andronov, Vitt, Haikin, Migulin, Krilov, Bogolyubov, etc. (Russia) [44–84].

From the viewpoint of the Theory of oscillations, oscillating systems can be classified in several big groups (classes):

1. Linear systems – they are described by linear differential equations, whose parameters (coefficients) are independent of the mode (regime) of work and of time.
2. Parametric systems – they are described by linear differential equations with periodic coefficients, i.e. the parameters are independent of the mode (regime) of work but dependent on time.
3. Nonlinear systems – they are described by nonlinear differential equations, whose parameters (coefficients) are time-independent but dependent on the mode (regime) of work.
4. Nonlinear-parametric systems – they are described by nonlinear differential equations, whose parameters are both mode (regime) and time-dependent.

The book presents an analysis of different oscillating systems belonging to the four major groups of the aforementioned classification. It is noteworthy that if one applies a rigorous approach, all modern oscillating systems should be referred to group four. But this would make the analysis extremely complicated, so in many cases either one or another set of relations and dependencies are ignored and the oscillating systems are referred to the other three groups. Moreover, it should always be remembered that the „linear” (or „linearized”) treatment may entail not only inaccuracies in the quantitative evaluations but qualitatively erroneous results as well. Besides, as a rule it is possible to generate and transform different signals only in systems of the latter three groups.

In accordance with the small signal theory and the filter method [78–85], the considered modulation-parametric systems, where small modulating signals and „pumping” high-intensity action are present, are referred to the second group even in the cases when nonlinear elements are used in the circuits.

Chapters 1, 2, 3, and 4 deal with effects and phenomena in modulation-parametric and nonlinear systems belonging to the second and third groups.

Chapters 5, 6, 7, 8, and 9 present a general theory of some phenomena taking place in group one and group two systems.

The systems belonging to group four are the most complicated and least investigated throughout the world. The phenomena occurring in the systems of this group may differ from the phenomena taking place in the linear parametric systems (group two) as drastically as any nonlinear processes, phenomena and systems differ from the linear ones. That is why, in principle, the fourth group of systems is boundless. A peculiar subclass of group four: modulatory nonlinear systems with adaptive self-control (kick-excited self-adaptive systems), is set apart in the book. Besides the typical nonlinear-parametric peculiarities, it also exhibits modulation interactions, whose intensity is small in comparison with definite basic oscillations in the system. The new initial conditions formulated for this subclass allow to qualitatively identify new oscillation phenomena and regularities in macro-systems.

Chapter 10 deals with the properties, regularities and phenomena in the separated subclass of systems, and together with Chapter 9 it tackles the phenomena of synergetic (adaptive) grouping of oscillating systems into stable formations.

Radiophysical systems undergo a new evolutionary stage today. While the first stage saw the development of the theory and practice of receiving and transmitting radio-engineering modules, and of some fundamental issues in the generation, amplification, modulation and manipulation of radio-engineering signals, the modern stage in the development of radio-engineering systems is marked by: the discovery and exploration of new physical effects and phenomena, the evolution of new analytical methods, the optimization of limiting characteristics, the design of qualitatively new precision sensors and radio-engineering means, the development of new methods of radio signal processing on the basis of adaptive and synergetic effects, neuron networks and the artificial intelligence principles.

The development logic of radiophysics can be depicted as a spiral of circles that are attached to each other, while at the same time, as a new turn is made, each subsequent circle rises to a higher level than the previous one.

As an outline this can be presented as the following sequence of stages:

- Discovery and exploration of new physical phenomena and effects, new theoretical generalizations;
- Evolution of analytical methods, synthesis and design of new sensors, devices and systems;

- Analysis and optimization of radio-engineering facilities and of their parameters and characteristics;
- Improvement of the element basis and a new circle of synthesis and analysis of radio-engineering systems and their characteristics;
- Emergence of a new set of physical phenomena and effects bringing about a new circle of synthesis, analysis and optimization of radio-engineering facilities;
- Interaction and mutual enrichment with other areas of science and technology – physics, electronics, mechanics, biology, chemistry, medicine, etc.

Figuratively speaking, the book is positioned on the circle corresponding to the exploration of new effects and phenomena in radiophysical systems, theoretical generalization, the development of analytical methods, and the implementation of general analysis of radiophysical devices and systems with a view to possible practical needs and applications.

The available literature in the field of Theory of oscillations can be conventionally divided into three groups.

Group *one* encompasses mathematical monographs, articles and other publications written by mathematicians, whose expertise is mainly in the area of asymptotic and qualitative methods of analysis. As a rule, these works are notable for the mathematical strictness and for the abstract nature of the methods put forward, while the constructive aspect of the research is only touched upon. For example, when assessing any methodological errors, the authors of this group of works consider only the possibility of elaborating such an assessment in principle, and not the particular procedure of its actual development that could be used in applied problems.

One can refer to group *two* such books, articles and other publications that belong to the other extreme – where authors are involved in the analysis of defined particular problems and come up with clear engineering and technical solutions.

It is obvious that somewhere „in the middle” one could distinguish a group *three* (a „buffer” one), containing works that develop theoretical aspects concerning specific types or classes of systems that can serve as a basis for experimental studies and for conducting the research in group two works. As a rule, the majority of the radiophysical and theoretical engineering elaborations are reflected in group three literary sources.

Due to its nature, the bulk of this book can be referred to group three but it also contains a number of elements belonging to group two.

Linear oscillating systems have been thoroughly studied. The employment of linear differential equations has allowed studying these systems in great detail and conducting an analysis entailing almost automatic application of the suitably developed mathematical technique.

When investigating parametric systems, one could utilize not only the specific technique developed by Floquet, Mathieu, Hill, Gorelik, Yakubovich, Starzhinsky, Taft, etc. [78–89], but almost all approximate methods evolved for analyzing nonlinear systems as well.

It is well-known that we are still in need of general methods for solving nonlinear differential equations, describing nonlinear and nonlinear-parametric systems. Considerable achievements in the theory of nonlinear oscillations were scored in the 1920s and 1930s, particularly in connection with the invention of the electronic (vacuum) valve and the intensive development of radio engineering. It is also worth nothing that the foundations of the mathematical technique adequate to the overall set of issues concerning the Theory of nonlinear oscillations were first laid out in Poincaré's classical works in celestial mechanics and in Lyapounov's elaborations on motion stability at the end of the 19th century [90–95]. A.A.Andronov [49] was the first to draw attention to the relation between these works and the task of exploring continuous oscillations in non-conservative systems. He was the first to introduce the term „self-oscillation”.

The rapid development of the modern Theory of nonlinear oscillations has led to the creation of a number of analytical methods that can be divided into two big groups:

A. Methods applicable to systems of both weak and considerable nonlinearity. The following are referred to this group:

1. The methods of the qualitative theory of differential equations developed by Poincaré and presenting geometric images (maps) of the different motions in the system by constructing a family of integral curves on the phase plane [90–92, 94, 96].

2. Methods of piece-wise linearization of nonlinear characteristics with subsequent solutions join in the border-line areas of the individual linearized parts [87].

B. Methods applicable to systems of weak nonlinearity: the small parameter method [54], the method of slowly changing amplitudes [51], asymptotic methods [97], quasi-linear methods [81, 82], the method of the harmonic balance [98], the energy method [99], the method of symbolic equations [63], the stroboscopic method [74], the method of M. Rausher [100], the averaging method [101], the quasi powered method [102], the variable scale method [103], the graph-analytical method [104], numerical methods [105].

The book develops an analytical technique for analyzing nonlinear and modulation-parametric phenomena in radiophysical systems, for investigating the conversion properties of radiophysical systems of a generator type. It also develops a modified complex amplitudes method for analyzing parametric and nonlinear systems. It proposes an adequate analytical technique for studying linear radiophysical systems with almost periodic parameters – linear spatial systems of algebraic equations. The concept of multi-index multiplication of spatial matrices is introduced and methods of conducting such multiplication are presented. A general analytical approach to the implementation of the principle of linear connection to an application of nonlinear radiophysical systems is also offered.

The fundamental research carried out in the past quarter of the century in the field of Radiophysics, Solid state physics and Low temperature physics revealed

radically new possibilities for utilizing the electric properties of a solid state for amplifying and transforming electromagnetic oscillations [4, 43].

The creation and utilization of parametric one-ports of different functional designation was a powerful stimulus for the development of modern Radiophysics and Mechanics [2, 78].

Van-der-Ziele [106], while investigating circuits containing nonlinear capacitance, first pointed out that such circuits can be used as low-noise amplifiers. Later on Vulle invented the semi-conductor nonlinear capacitor [107], and Sul offered a microwave solid state amplifier based on ferrite [108].

Nowadays, parametric amplifiers and converters based on the nonlinear capacitance of a semi-conductor diode are widely applied in various areas of science and technology. The developed parametric systems cover an enormous frequency range – from single-digit hertz frequencies to those corresponding to waves in the sub millimeter band [85].

A number of monographs [78, 81–85, 107] and a multitude of articles published in periodicals explore parametric systems.

Most generally, parametric systems can be divided into three major classes: frequency-sustaining systems (of a communicating and reflective type) and systems transforming the frequency „upwards” or „downwards” along the frequency axis. As the frequency is transformed „upwards”, it is possible to achieve amplification even without regeneration.

When a weak signal enters parametric systems of a regenerative type, it draws energy from a pumping source and its intensity increases. The mechanism of establishing a positive inverse feed-back in parametric systems is conditioned by the reciprocal properties of the linear-parametric circuits. When the resonance systems serving as a load for the combined frequency components are tuned in a particular way, the pulsation of the reactance with pumping frequency leads to the introduction of energy in the signal circuit. In the course of the analysis, this property of parametric systems is reflected through a negative resistance introduced into the signal circuit. Obviously, when selecting the load of the combined components in a particular way, it can be introduced into the circuit of the signal not only negative or positive resistance but negative reactance (negative capacitance or negative inductance) as well. When a broad-band negative equivalent reactance introduced into the signal circuit is ensured with respect to a signal spectrum, the sensitivity of radiophysical systems can be substantially promoted by compensating the own or parasitic reactive component of their impedance [78].

One of the promising directions in the field of parametrics is that of parametric systems with four and more working frequencies [51, 78, 85, 109] using differential and sum combined frequencies obtained as a result of mixing the signal frequency and the pumping one. We term such systems parametric modulation systems (PMS) since, on the one hand, the manipulations are carried out with a power accumulating (parametric) element and, on the other hand, all regularities of modulation influence are in existence. PMS are based on the principle of modulating forced oscillations

in a resonance system containing a nonlinear (or time-dependent) reactive element. It is worth noting that any nonlinear oscillating circuit, where forced oscillations or self-oscillations (the latter in the case if it is a part of a generator system) have been excited, is a modulation-parametric system with respect to a signal with a spectrum falling far below the transmission band of the circuit.

The fundamentals of the theory and the design of four-frequency parametric systems are presented in the works of Ekhardt, Stetser, Bayar, Hauson, Kuh, Mathey, DeJager, M. D. Karasev, M. E. Gertsenstein, V. S. Etkin, Yu. L. Khotuntsev, V. N. Detinko, etc. [78].

An interesting property of four-frequency parametric systems from a physical point of view is that, given definite PMS working modes, the active and reactive parts of the input differential impedance of the system have a negative sign within a broad frequency range of the input signals up to d. c. [78]. This opens up broad vistas for creating regenerative video-amplifying systems without frequency transformation, low-noise parametric frequency correctors, etc.

The terms of „negative capacitance” and „negative inductance” were introduced by Van-der-Paul B. [48], who developed the first circuits for their implementation on the basis of negative resistances. Tube and transistor feed-back circuits also allow realizing negative capacitances and inductances, which, however, come out as narrow-band and frequency dependent ones with relatively high own fluctuation noise.

On the whole, the negative resistances used in science and engineering can be conventionally referred to the following major groups:

1. „Natural” negative resistances typical of the devices and elements with falling volt-ampere characteristic (VAC), in both static and dynamic mode (electric arc, dynatron, transitron, thermistor, diistor, tunnel diode, etc.).
2. Negative resistances based on the period of electron drift. A falling VAC is realized only in a dynamic mode (diodes with field emission, klystrons, magnetrons, avalanche-drift diodes, Gunn diodes, etc.).
3. Negative resistances based on phase shifting with an external active three-pole element. A falling VAC can exist in both static and dynamic modes.
4. Negative resistances realized with the help of a feed-back. A falling VAC emerges only in a dynamic mode. This method is particularly important from a practical point of view, since it serves as a basis for designing tube and semiconductor generators, regenerative filters, etc.
5. Negative resistances resulting from parametric interactions with the participation of a power-accumulating (reactive) element.

It is obvious that negative resistances are proper not only to individual elements but to devices (semiconductor, ferromagnetic, ferroelectric, etc.) and systems (generator, parametric systems, etc.) as well.

Nonlinear resonance systems with semi-conductor diodes are widely applied in various radiophysical devices – low-noise parametric amplifiers, generators, converters, highly stable parametric frequency multipliers and dividers, frequency

modulators, receivers with electronic tuning, trigger circuits, parametrons, etc. [63, 78, 85, 109, 110].

As regards the utilization of modulation-parametric interactions for the effective control of equivalent impedances, a number of issues emerge: first, with respect to the power interchange between pumping circuits and signal in parametric systems in different specific modes and under varying conditions; second, the specificities of the nonlinear and parametric resonance in resonance systems with partially forward-biased (forward-conducting) $p - n$ junction; third, the legality of using small-signal differential parameters and the quasi-static VAC of $p - n$ structures in the conditions of high frequencies and amplitudes, etc.

Generators in autonomous and non-autonomous modes have been widely investigated and used as frequency converters in recent years [78]. This interest has been provoked mostly by the possibility of coming up with high conversion ratios, and it is also related to the development and investigation of autodyne systems for short-range Doppler radars, as well as with the extension of the functional capacities of generator systems. Analysis shows that modulation-parametric phenomena, analogous to those taking place in parametric systems with external pumping, can be manifested in non-autonomous generator systems. A regenerative change in the effective (equivalent) impedance is observed in the synchronization band and, therefore, the synchronized generator can be viewed as a one-port with controllable parameters with respect to external video-signals. A generator nonlinear one-port, set in a conversion mode, may have negative conductance at intermediate frequency, whereas the module of its value reaches its maximum at a power of the basic high-frequency (or SHF) oscillations smaller by far than the possible maximum. The mechanism of this effect is also related to modulation-parametric and detuning effects.

The interest in the investigation of nonlinear parametric interactions of oscillations of different frequencies in complex generator systems that has risen in recent years is also related to the possibility to create, on their basis, selective receiving systems, capable of separating signals of close frequencies and comparable amplitudes [78].

Short-range Doppler radars with self-mixing diodes (Doppler autodynes) are among the most widely developed and applied functional generator systems. They are miniature receiver-and-transmitter modules, widely used in systems for short-range radiolocation [111]. Their major advantage is their compact form and low consumption. Besides, their small transmission power is particularly topical in relation to the problems of electromagnetic compatibility. Certain issues emerge in this connection regarding: a more detailed analysis of the conversion properties, of the spectrum of oscillations, of the stability of Doppler autodynes; the optimization of their characteristics and the development of new functional solutions. For example, one of the ways of increasing their sensitivity is to use one element for generating the oscillations that are to be emitted and for amplifying the received signal, and to use an external converter for the frequency conversion aimed at taking

down (recording) the Doppler signal [78]. Moreover, the effects of modulation-parametric amplification can be used for increasing the general sensitivity of the system. The fundamental works on Doppler autodynes (for example [111]) solve the radiolocation problem of continuous signals mainly by taking into account the first harmonic in the Doppler spectrum and the amplitude modulation of the oscillations. Considerable interest in the structure of the signal in Doppler autodynes has been evinced in recent years in connection with the investigation of the utmost parameters of the facilities for short-range radiolocation and extension of their functional capacities [78]. It becomes necessary to analyze in greater detail the spectrum of the oscillations in Doppler autodynes by taking into account both amplitude and phase modulation with the first and second harmonics of the Doppler frequency shift of the reflected signal.

Science and engineering have been using radiophysical systems of growing complexity. One can juxtapose an equivalent circuit to each such system, consisting of a definite number of sources of electromagnetic power, inductances, capacitances, active resistances, connected in a certain way. These circuits with point parameters may be classified in a definite way and the common properties of the separate classes may be worked out.

Above we adopted a classification of the radiophysical systems with point parameters dividing them into four classes – linear, parametric, nonlinear and nonlinear-parametric.

The first class is conditioned by the multitude of systems with time-independent parameters. The first subclass can be identified as containing autonomous linear systems with constant parameters and driving forces (the latter may even be omitted). Non-autonomous systems with constant parameters and time-dependent driving forces can be referred to a second subclass.

The second class encompasses systems whose parameters are explicitly time-dependent. This class can also be subdivided into two: a subclass of linear systems whose parameters change in time, and a subclass of systems containing explicitly time-dependent nonlinear parameters. We separate the second subclass here, since the phenomena and the typical regularities occurring in numerous practical cases, such as, for example, modulation-parametric systems or systems to which the small signal theory applies, are similar to those in the class of linear-parametric systems.

In the general case, nonlinear-parametric systems constitute the most general and extensive fourth class of systems, including the remaining three classes as particular cases.

The topicality of the developing theory of second-class radiophysical systems in the possibly most general form is confirmed by the present-day condition of science and technology: 1. The success scored by solid state and cryogenic electronics has led to the appearance of new radiophysical elements; a new branch – bioelectronics – has also emerged; 2. A precise analysis of a broad range of universal, generator and converting radiophysical systems is only possible if they are considered as second-class systems with parameters changing in time; 3. The behaviour of the

radiophysical systems in the majority of the practical cases is determined by the „resonance” phenomenon which has been insufficiently investigated with respect to second-class systems, including the general problems of stability.

In this connection, the task of developing a general method for analyzing second-class radiophysical systems that would be convenient for practical research, and of assessing the potential capacities of these systems has been formulated. The basic mathematical technique for investigating second-class systems presented in this book is the developed version of the method of complex amplitudes, which ranks closest to engineering practice.

The linear connection principle [112] is used as a basis for offering a method for analyzing nonlinear systems by using linear methods.

Systems described by differential equations of the second or any higher order with periodic coefficients have been tackled in a lot of works: beginning with the classical ones of Lyapounov [93] and ending up with the numerous publications by modern researchers, for example [78–89] and many more. In spite of the considerable number of instructive mathematical publications, the problem of analyzing qualitatively the free processes in a parametric oscillating circuit cannot be regarded as solved. There is an essential difference between the analysis of the abstract mathematical equation and the particular engineering-and-physical system. As a rule the engineering-and-physical problem consists of three parts. The first one allows using the physical properties of the system as a starting point to obtain its schematic and analytical description, as well as the respective mathematical equation. The second part consists in solving and exploring the equation obtained. The third part provides a physical-and-engineering interpretation of the results. The mathematical problem is a component of the engineering-and-physical one and constitutes the latter’s second part. The powerful mathematical means used in its solution often allows obtaining in-depth results. Thus, in a certain sense, the engineering-and-physical approach is broader than the mathematical one, but the latter is more profound. When solving the engineering-and-physical problem, it is important to adapt and use adequately a relevant mathematical technique. The book seeks to combine the general formulation of the engineering-and-physical problem concerning the processes in a generalized oscillating circuit with the profundity of the mathematical exploration.

The general analysis of the stationary modes of oscillating systems with periodic parameters is of great theoretical and practical significance. Similar obstacles crop up in connection with many problems related to the Theory of nonlinear oscillations, in particular when investigating parametric amplification and generation of oscillations, frequency modulation, detection and conversion, suppression of undesired oscillations and intermodulation distortions, etc.

The phase plane is of considerable importance for the qualitative analysis of the free processes in a linear oscillating circuit (either autonomous or non-autonomous) with periodic parameters. We shall use the phase plane as a basis for a visual presentation of a dynamic picture of the multitude of free processes in

non-autonomous periodic oscillating circuits. The possible transformations of the equations of an oscillating circuit with periodic and almost periodic parameters will be given and the expedience of using different equation forms will be analyzed. A qualitative picture of the free processes in an oscillating circuit will be presented on the basis of the mathematical theory of Hamiltonian systems.

Oscillating circuits with periodic parameters can be divided into two groups. One of them includes oscillating circuits where, given arbitrary initial conditions, free processes are limited. The other group, respectively, encompasses oscillating circuits, whose initial conditions can be selected in such a way as to ensure unlimited free processes. Each group of oscillating circuits, in its turn, is characterized by a set of stability and instability areas. It is particularly important to develop analytical approaches for determining the area of stability or instability to which the specific oscillating circuit belongs.

The „resonance” phenomenon occupies a peculiar position in natural and other sciences, in technology, in civil engineering subjects, in medicine, in the theory of musical instruments, in aeronautics theory, in rocket technics and in astronautics, etc. [4, 44-78, 113-116]. Resonance is often manifested in the world that surrounds us either as a highly useful phenomenon or as an extremely harmful one. Radio communications, radio broadcasting, television and the other radio engineering systems would be absolutely inconceivable without resonance. This book shows that resonance is quite multiaspectual and multiform even in oscillating circuits with constant parameters. Resonance phenomena in nonlinear oscillating systems are virtually boundless.

Linear systems are quite frequently identified as systems with constant parameters not only in textbooks and other teaching aids but in scientific works as well. This approach reduces drastically the class of linear systems, since it excludes linear systems with time-dependent parameters. The principle of linear connection, formulated relatively recently, has boosted the significance of linear systems with variable parameters, since it follows from this principle that if the whole set of linear systems can be studied, this will automatically lead to the establishment of the necessary scientific basis for investigating the processes occurring in nonlinear systems.

The theory of parametric resonance, created mostly by G. S. Gorelik and the school of L. I. Mandelstam and N. D. Papaleksi [46, 47, 50, 86, 115, 118], has been recognized and renowned throughout the world namely because it substantially broadens our notion of the resonance phenomenon. It has been proved that sine functions, to which harmonic resonators respond, are not „the simplest” oscillations. There is quite a broad class of functions which elicit response from resonators whose parameters change periodically in time.

The Theory of nonlinear systems has by now reached a high level of development. The objective difficulties related to the establishment of a general analytical basis uniting nonlinear systems are well-known. While the superposition principle is valid for the Theory of linear systems, as it brings together all possible approaches

for their analysis, no similar general concept has been identified with respect to the Theory of nonlinear systems. Due to this, the Theory of nonlinear systems is in a sense a theory of particular cases. The relation between the own, free and forced oscillations in nonlinear systems is quite diluted and cannot always be identified. Even if the process is known under certain initial conditions, one cannot always use it as a basis for stating something definite about the oscillating process under other, even quite close, initial conditions. Hence, qualitative methods of analysis that allow of identifying the essential properties of the oscillating processes without going into great detail acquire particular significance in this situation.

Nonlinear systems have many distinctive features which hamper their classification. There are two large classes that are generally accepted: autonomous and non-autonomous systems. The latter may be non-autonomous due to the action of an external signal, and also due to a time-dependent change in the parameters of the system. The systems of the latter subclass contain the easily distinguished (according to their practical significance) set of nonlinear systems whose parameters are obviously time-dependent and governed by a periodic or almost periodic law.

The method of complex amplitudes can be adapted for analyzing nonlinear systems. Its application allows a considerable simplification of the intermediary analytical transformations. But this method has a specificity: it clearly distinguishes positive from negative frequencies, though they are physically inseparable. Hence, an artificial unification of positive and negative frequencies within the method of complex amplitudes can lead to mistakes. Frequently it is expedient to use the method of complex amplitudes for the intermediate analytical transformations and then to switch to another method for the rest of the analysis.

The most typical property of any oscillating circuit with constant parameters is the fact that a sine action exercised in a stationary mode leads to a sine response, i.e. a sinusoid gives rise to another sinusoid. That is why the amplitude vrs. frequency characteristic of an oscillating circuit together with its phase vrs. frequency characteristic represent a complete description of the circuit adequate to the processes occurring in it. In the case of a nonlinear oscillating circuit, one can imagine that a single sinusoid engenders an infinite set of sinusoids. No characteristic obtained by accounting only for the first harmonic can be regarded as a complete characteristic adequate to the processes in the nonlinear circuit. In principle, there can be nonlinear circuits whose response to sine excitation may not contain a first harmonic at all. Hence, the purpose of developing the theory of resonance in a nonlinear oscillating circuit is to come up with a general approach covering the whole real spectrum.

The processes of self-organization and adaptive grouping in stable formations are an object of analysis not only in the living world but also in technical systems, where they are related with different phase and bifurcation transitions and with a specific response of the system to external action [32-38, 42, 78, 117]. At present these issues are referred to the new scientific domain of Synergetics [32], and more

generally to the theory of self-organizing systems. An essential role in the self-organization and adaptive grouping in radiophysical and electromechanical systems is played by the reversibility of the interactions, by the bifurcation nonlinearities and cooperative effects when feedbacks are created [32–38, 78]. A primary form of self-organization in the non-living world is the spatial grouping of separate particles (component parts, subsystems) into more complex systems with definite stability. In addition, the following important situation occurs: the objects grouped together into stable formations are, as a rule, oscillating systems.

Nowadays the interest of an enormous number of researchers from different scientific areas and schools is attracted by the issues of self-organization of matter and the emergence of regular motions, by the processes of adaptive maintenance of definite, virtually unchanging, parameters of the systems grouped in stable formations, by the adaptive adjustment of the systems to external action, etc. [32–38, 42, 78, 117]. The issue of the essence and mechanisms of the grouping of the elementary material objects (microscopic and macroscopic systems) into stable formations is related to one of the most topical and important tasks of modern science. To one extent or another, all of the following can be referred to the „history” of the issue under consideration: the forecast of the atomic composition of matter, the discovery of the periodic system of elements, the rise and evolution of the ideas of electric, oscillating, wave and quantum phenomena, etc. One should mention here the experiments and the scientific treatises of R. Hook (1635-1703), G. Mechtenberg (1744-1799), M. Faraday (1792-1867), V. Berknes (1862-1951), J. Ch. Bousse (1859-1937), P. N. Lebedev (1866-1942), B. B. Golitsin, S. A. Boguslavskiy, M. I. Mandelstam, N. D. Papaleksi [44-53, 118]. The works of the classics listed above lay out the foundations of the theory of periodic motions, of ponderomotive interactions of different oscillating systems. A. Einstein made wide use of oscillating analogues to describe the heat-consuming property and solidity of substances. It is worth noting that in spite of the availability of a number of wonderful models (those of Gins, Tyuring, Eigen, Zhabotinski-Belousov and many more), the modern stage in the development of synergetics as a scientific area is related to the analysis of most simple basic models, which serve as a foundation for exploring some general features of the processes of adaptive grouping and self-organization.

This book has the task of studying definite synergic aspects of oscillating systems with a limited number of degrees of freedom. Some processes of ponderomotive equilibrium and continuous oscillations in grouped systems are examined. A new subclass of systems is formed, i.e. modulation nonlinear parametric systems with adaptive phase self-adjustment. Figuratively speaking, objects that fall into groups and share a property designated as $1 + 1 \neq 2$ are studied. In other words, two grouped objects have new properties that neither of them possesses separately. In the case of radiophysical systems, the properties regarded as most important are the adaptive self-tuning of the system as a whole, as well as the consistency of some internal characteristics and parameters of a separate element under changing external actions.

M. Plank [119] dedicated a cycle of his works to the issue of the absorption and radiation of electromagnetic waves by a system of linear resonators. He believed that in the case of irradiation with an electromagnetic wave the latter changes its resonance properties itself, thereby providing „multiple resonance”, i.e. one and the same source of electromagnetic waves excites quite different resonators (as regards their own frequency). A result achieved in this book shows that the „multiple resonance” mechanism lies neither in the properties of the excited resonators, which can be strictly linear, nor in the emitters – it lies in the nonlinearity of the very process of interaction between the electromagnetic wave and a separate resonator. Moreover, a phenomenon of „quantization” of the possible amplitudes of the oscillations excited in each resonator, taken separately, is found.

The basic synergetic system for analytical purposes is the system of two interacting electric oscillating systems with a mechanical degree of freedom. The conditions for adaptive grouping of the system in stable formations are studied.

A class of modulation nonlinear-parametric systems and phenomena with adaptive self-tuning of the phase is created. For the sake of brevity they are termed „argument systems (phenomena, oscillations)” or „an argument method of oscillation excitation”. This class corresponds to new initial conditions: nonlinear or linear oscillating systems experience the action of external periodic forces that are nonlinear as regards the coordinate of the excited system. The new initial conditions determine a specific action on the argument of motion in the system, which hunts with respect to the argument of speed and acceleration. For example, a linear differential equation of argument oscillations may be written in the form

$$\ddot{x}(t) + 2\beta\dot{x}(t) + \omega_0^2 x(t + \Delta \sin pt) = 0.$$

Obviously, the argument processes are described by equations that cannot be transformed into equations with periodic coefficients, which are typical of parametric phenomena, i.e. into Mathieu’s and Hill’s equations. In fact these are equations with constant coefficients and a specific nonlinear right-hand part.

Given a non-zero amplitude Δ of the argument external force, over a certain part of the period $\frac{2\pi}{p}$ the argument of the shift along the coordinate will run ahead of the argument of acceleration, while during the remaining part of the period it will lag behind that argument. Under certain conditions, a predominant delay of the coordinate argument with respect to the argument of acceleration may occur in various oscillating systems (radiophysical, electromechanical, etc.), which may lead to the excitation of continuous oscillations.

The foundation for studying these processes is the analysis, from the general positions of the Theory of nonlinear oscillations, of the processes and phenomena well-known in Radiophysics, Electromechanics, Microwave electronics, Optics, technology for accelerating charged particles, etc. [4, 42, 69, 78, 120–149]. The general regularities identified during the analysis allow for a model consideration

of processes with identical properties in relatively simple oscillating systems with a limited number of freedom degrees, for formulating a definite mechanism for exciting stable continuous oscillations, for substantiating – both theoretically and experimentally – a number of new, previously unknown but objectively existing, properties and regularities of the oscillating systems.

The mechanism of exciting natural or close to natural oscillations through the action of an external high frequency force allows obtaining stable periodic regimes with high multiplicity of unitary frequency conversion, high efficiency, a discrete set of possible amplitudes, self-control of the input of energy from the external source, high stabilization coefficient in the case of changing external action amplitude and load of the oscillating system in a very wide range.

The model examination of the processes of wave interaction with oscillators allows to create a method of converting waves from the light band into submillimeter waves. This issue is directly related to the problems of the controlled thermonuclear reaction.

The book has the following structure:

Chapter 1 formulates a principle of reversibility of the modulation-parametric interactions which underlies the effective control of equivalent impedances in radiophysical systems. This serves as a basis for a classification of the radiophysical systems containing forces aimed at changing the effective reactive parameters and the dissipation in these systems. The classical Manley-Row energy relations are generalized for an arbitrary number of signal sources in the conditions of linear and periodic in time reactance, or nonlinear reactance with explicit time dependence. A model of a modulation-parametric modulator and a generalized conversion matrix are presented. An analytical technique for investigating modulation-parametric, conversion, autodyne and other processes and modes in autonomous and non-autonomous generator systems under the action of small external signals is developed.

Chapter 2 presents a theoretical generalization of the modulation-parametric interaction in nonlinear and parametric oscillating systems with external pumping and generator oscillating systems. Nonlinear and modulation-parametric phenomena occurring in several classes of radiophysical systems – four-frequency modulation-parametric systems, generator non-autonomous systems, etc. are described. It is shown that under certain conditions, modulation-parametric interactions lead to an effective change in the value and sign of the frequency-determining (reactive) parameters and of the dissipation, and to the appearance of new conversion properties and regularities, etc. Methods for controlling impedance parameters in modulation-parametric systems with external pumping and non-autonomous generator systems are developed. A number of effects occurring in generator radiophysical systems with synchronizing and asynchronous external action are presented, they are analyzed theoretically and methods for signal amplification, conversion and processing are offered. The peculiarities of the modulation-parametric interactions as reflected in the conversion properties,

spectrum and stability of microwave autodyne systems for short-range radars are studied. Autodyne systems described by differential equations of the second and third order are analyzed in theoretical and comparative terms. Different applications of one-ports with negative parameters based on four-frequency parametric systems and non-autonomous generators are presented, i.e. modulation-parametric devices in systems for optical television and radio engineering reconnaissance, vidicons for security purposes, pyroelectric receivers for special fire-warning systems, special sensor systems, including an inductive one for analyzing the vibrospectrum of the body and other elements of machines, aircraft, marine vessels, railway and motor vehicles.

Chapter 3 conveys the investigation of nonlinear and parametric resonance in a generalized radiophysical oscillating circuit. A generalized modified method of complex amplitudes for analyzing processes in nonlinear radiophysical oscillating systems in a general form is offered. Some issues related to the realization of parametric one-ports are considered. A theory of diffusion impedance and detection with a semi-conductor diode in the case of high frequencies and increased signal amplitudes is presented. The influence of the effect of charge accumulation by non-basic carriers on the diffusion conductance and frequency characteristic of the detection with a p-n junction is investigated. So is the latter's work in resonance systems under the real conditions of a periodic signal with large amplitude, at high frequencies and with various loads.

Chapter 4 considers issues of the chaotization of oscillations in radiophysical systems. Some basic concepts, such as bifurcation, chaos, strange attractor, fractal dimension are presented. The conditions and basic mechanisms of transition from determined to chaotic oscillations are described. The basic methods and criteria for identifying chaotic oscillations are provided. The conditions for the emergence of specific instabilities are theoretically considered as stochastization of the oscillations in generator systems with additional delaying feedback without any random actions. It is shown that oscillation chaotization with complete loss of the informative qualities of the system is possible in the case of a system with delayed self-action, such as an autodyne system for short-range radars.

Chapter 5 develops a version of the method of complex amplitudes suitable for analyzing linear systems with periodic and almost periodic parameters. It is shown that this method can be used to analyze not only the stationary mode but the transition process as well – the concept of complex frequency is employed to this end. Properties of the elements of radiophysical systems, such as time-dependent periodic and almost periodic resistances, capacitances, inductances, are considered; their energy characteristics are studied. The mechanism of power transformation with reactive and active elements is investigated.

Chapter 6 presents the resonance properties of a generalized non-conservative oscillating circuit with constant parameters. It is shown that the „resonance” phenomenon refers not to the circuit but to a definite physical value: capacitor charge, magnetic flux of an inductance, etc. When the circuit is considered as a

one-port, the concepts of amplitude resonance and phase resonance are used. The energy balance in an oscillating circuit with constant parameters is analyzed.

Chapter 7 contains formulations of general theorems for systems with positive elements concerning the relation between the parameters of the system and the matrix elements of the respective differential vector equation. Infinite systems of algebraic equations for some typical systems with periodic parameters are obtained; the properties of these systems of equations are explored, the general case relation between the complication of the radiophysical systems and the respective alteration of the systems of equations describing them is identified. Solutions of the infinite systems of algebraic equations are constructed in the form of infinite continued fractions. A clear dynamic picture of the multitude of free processes occurring in an oscillating circuit with periodic and almost periodic parameters is given. Theorems concerning the conditions that would be sufficient for the stability or instability of an oscillating circuit with periodic parameters (parametric oscillating circuit) are formulated. The phenomenon „resonance” in a linear oscillating circuit with periodic parameters is analyzed. A generalized interpretation of parametric resonance is provided by using visualizing geometrical categories. The equation of the parametric oscillating circuit is correlated with an expanded hyperplane of the solutions, forming a four-dimensional Euclidean space provided with an oxygonal coordinate system. Linear systems with almost periodic parameters are examined and the concept of „branching” of almost periodic functions is introduced. It is characterized by a positive integer depending on the number of the incommensurable frequencies in the spectrum. An adequate analytical technique for studying linear radiophysical systems with almost periodic parameters is presented. The concept of spatial matrixes is introduced and methods of multi-index multiplication of space matrixes are provided. A classification of linear systems with almost periodic parameters by the respective multi-dimensional infinite systems of linear algebraic equations is offered.

Chapter 8 offers a general analytical approach to the realization of the principle of linear connection as applied to nonlinear radiophysical systems. The phenomena of „strong” and „weak” resonance in a nonlinear system with explicitly time-dependent parameters are considered. It is shown that the output power spectrum of the microwave generators, where nonlinear interelectrode capacitances are connected to resonator systems, largely depends on the number of the degrees of freedom in them. If the number of the degrees of freedom is two or more, the oscillations in the generator are almost periodic and their branching capacity increases.

Chapter 9 presents a generalized theory of adaptive (synergetic) grouping of coupled resonance systems into stable electromechanical formations. An oscillating system with three degrees of freedom is studied. It consists of two linear oscillating systems coupled through a magnetic and electric link and capable of changing the distance between them. The generalized conditions for stability of the coupled oscillating systems are studied on the basis of a system of three differential

equations describing the electric and mechanical processes in them. A physical effect consisting in an unambiguous correspondence between the frequency of the exciting generator and the relevant stable state of the system (including configuration and resonance properties) is presented. A system of two linear oscillating circuits linked with a capacitor, one of whose plates can be shifted freely, is separately explored. It is shown that the position of the mobile plate is unambiguously determined by the frequency of the exciting generator connected to one of the coupled oscillating circuits, i.e. that the capacitor „follows” the frequency of the generator by adequately changing the distance between the plates. An expression for the ponderomotive force seeking to change the position of the mobile plate in relation to the fixed one is obtained. The condition for a zero ponderomotive force is used as a starting point for establishing the dependence between the stationary values of the linking capacitance and the distance between its plates as a function of the frequency. A theory of resonance connection frequencies (natural resonance frequencies) of a radiophysical system with 2.5 degrees of freedom in the conditions of adaptive grouping of the system in a stable electromechanical formation is developed. An effect of „frequency attraction” is described. It differs, both in quality and in principle, from the effects of frequency entrainment and synchronization widely known in Radiophysics and Mechanicns.

Chapter 10 presents a theory of the newly-formed class of modulation nonlinear-parametric phenomena and systems with adaptive phase self-tuning (kick-excited self-adapting systems). The theory includes two major elaborations: a general analysis of the dynamic properties and regularities in a linear or nonlinear resonance system under the impact of a nonlinear-on-the coordinate external periodic force and a linear or nonlinear resonance system under the action of a falling electromagnetic wave. A phenomenon of excitation of continuous oscillations with a discrete set of stable amplitudes in linear or nonlinear oscillating systems under the action of an external periodic force, nonlinear on the coordinate of the system subject to excitation, is described. This is a peculiar „quantization” by the parameter intensity in a macro system. The phenomenon has a high degree of generality and besides radiophysical systems, it also occurs in various other material media of an oscillating nature. The novelty of the phenomenon is based on the novelty of the premises presupposing the existence of an external exciting force, nonlinear on the coordinate of the system subject to excitation. The new premises predetermine the qualitatively new characteristics of the phenomenon: „quantization” (discreteness) of the stationary amplitudes, adaptive self-sustenance of the oscillations in the situation of external and internal changes in parameters and impacts, etc. Some analytical dependencies determining the discrete set of stable amplitudes for both nonlinear and linear oscillating systems are derived. It is shown that the key condition for rendering the possible stationary amplitudes discretization is the nonlinear dependence of the external impact force with respect to the coordinate of the system subject to excitation. Furthermore, the system tends towards a stationary mode with a phase belonging to a discrete set of favourable

phases. This is the gist of the adaptive nature of system-grouping in a discrete series of stable dynamic states, adaptive tuning and sustenance of oscillations with preset kinematic parameters. The fundamental investigations are based on a model nonlinear differential equation describing a number of real systems, such as simple pendulum, Josephson superconducting junction, oscillating motions of space particles, etc. Special emphasis is laid on the „quantization” of the oscillations in an oscillator under the action of a falling electromagnetic wave. Data from the numerical and natural experiment exploring the characteristic regularities of the phenomenon are quoted. A diagram involving multiple bifurcations for the attractor set of the system under consideration is obtained and analyzed. The complex dynamics, evolution and the energy boundaries of the multiple attractor basins in state space corresponding to energy and initial phase variables are obtained, traced and discussed. An analytic proof is presented showing the existence of „quantized” oscillations for the kick-excited pendulum. An analytic approach is given applicable to the cases of small and large amplitudes (small and large nonlinearity). The spectrum of possible oscillation amplitudes for the pendulum is studied as well as its motion in a rotational regime under the influence of an external non-homogeneous periodic force. Generalized conditions for the excitation of pendulum oscillations under the influence of an external nonlinear force are derived. A generalized model of an oscillator, subjected to the influence of an external wave is considered. It is shown that the systems of diverse physical background, which this model encompasses by their nature, should belong to the broader class mentioned of „kick-excited self-adaptive dynamical systems”. Derived also are generalized conditions for the transition of systems of this „oscillator-wave” type to non-regular and chaotic behaviour. For the purpose of demonstrating the heuristic properties of the generalized „oscillator-wave” model from this point of view are considered the relevant systems and phenomena of the kick-rotator, quantized cyclotron resonance and the mega-quantum resonance-wave model of the Solar System. We point to a number of other natural and scientific phenomena, which can be effectively analyzed from the point of view of the developed approach. In particular we stress on the possibility for development and the wide applicability of specific wave influences, for example for the improvement and the speeding up of technological processes. A number of practical applications of the method of exciting „quantized” oscillations in macro systems are presented.

Due to the considerable volume of the material in the book, it is impossible to detail elaborately all the results concerning the issues under consideration. Hence a number of issues as well as some applications are only most broadly outlined and in such cases relevant works of the author are quoted. The purpose is to focus on the theoretical generalizations and on the new theoretical points, presenting them most comprehensively and in a logical sequence. The range of references is sufficiently extensive to give the reader an idea of the scale of the investigations carried out in this and other related areas in recent years, moreover one should bear in mind that modern phenomena and methods are invariant with respect to the material carrier

and that they are elaborated in the sources from the perspective of different though close scientific disciplines.

The different paragraphs in the book, as well as all mathematical expressions, tables and figures have two-digit numbers indicating the chapter they belong to.

The denotations used in the book are not listed separately in the beginning since the enormous volume of the material, as well as the relative independence and the peculiarities of the different chapters do not allow treating these denotations in a systematic and uniform manner. Wherever possible, we have used universally accepted denotations.

Not having an usual practice of English, the quality of the English of this book is certainly affected. The reader may excuse this fact.