

PREFACE

This book contains the lectures we gave in the physics department at the Torino (Italia), Cosenza (Italia), Ufa (Russia) and Maringá (Brazil) Universities. The course was mainly devoted to graduate students.

In the first chapter, the variational calculus is presented, with a special emphasis on the case in which the boundary conditions follow from a surface contribution to the total functional. The connection of the variational calculus with the principle of virtual works is discussed in detail.

In the second chapter, we discuss the theory of elasticity of nematic liquid crystals. This theory was developed by following a procedure similar to the one used in the classical books of elasticity for solid materials. In the first part of this chapter, the stress tensor is written in terms of the nematic director \vec{n} . In the second part, in terms of the tensorial order parameter \mathbf{Q} . The temperature dependence of the elastic constants is deduced in a simple way by means of a Landau-like expansion of the elastic energy in a power series of the spatial gradients of \mathbf{Q} . The elastic behavior of nematic materials in the presence of external fields is also analyzed. In particular, we discuss the flexoelectric and orderelectric polarization connected to the mechanical deformations of the nematic liquid crystals. The extension of the usual theory of elasticity, when the second order spatial derivatives of the tensorial order parameter are used to define the stress tensor, is also discussed in this chapter.

In the third chapter, we present simple applications of the theory of elasticity for nematic liquid crystals. We analyze, among other applications, the usual Fréedericksz transition, in the presence of elastic- or solid friction-like boundary conditions. More complicated cases, like a hybrid cell submitted to an external electric field perpendicular to the initial deformation plane, are also presented, mainly for their pedagogical importance.

The molecular models and their limits are discussed in the fourth chapter. There it is shown how it is possible to evaluate the elastic constants when the interparticle interaction is known. We consider the classical interaction laws of Maier–Saupe, Nehring–Saupe and quadrupole–quadrupole. The theory is then applied to a lyotropic nematic liquid crystal doped with ferrofluid particles. In this chapter, we discuss also the presence of linear terms in the first and second order derivatives in the elastic energy density of a nematic liquid crystal. We show that terms of this kind exist, but they are originated by the free energy density of the reference state.

In the last chapter, the spontaneous deformation localized near the limiting surface is discussed. We analyze the origin of this subsurface deformation in the framework of a step-like distribution of density and of scalar order parameter. The study is performed also by means of pseudo-molecular models. This chapter is a little bit more complicated than the previous ones, and it contains mainly the papers we have published on the subject. It represents, in a sense, the point of view of the Italian school on the K_{13} problem.

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