

CONTENTS

PREFACE	ix
1. VARIATIONAL CALCULUS	1
1.1. Introduction	1
1.2. Standard Variational Problem: Strong Anchoring	2
1.3. Standard Variational Problem: Weak Anchoring	7
1.4. Mechanical Interpretation of the First Variation of a Functional	11
1.5. Second Variation and Jacobi Equation	13
1.6. Well-Posed Problem: Different Approaches, Same Solution	17
1.7. Functionals Containing $\phi''(z)$: Strong Anchoring	20
1.8. Functionals Containing $\phi''(z)$: Weak Anchoring	23
1.9. Functional with Discontinuous Extremizing Function	27
1.10. Ill-Posed Problem: Different Approaches, Different Solutions	30
1.11. Functionals Depending on Two Functions $\phi(z)$ and $\psi(z)$	34
1.12. The Junction Problem	38
1.13. Generalized Junction Problem	42
1.13.1. First example	42
1.13.2. Second example	43
1.13.3. Third example	44
1.13.4. Fourth example	47
1.13.5. Fifth example	48
1.13.6. Conclusions	51
1.14. Functional $F[\phi]$ in the Three-Dimensional Case	51
1.15. Dirichlet Problem	53

2. THEORY OF ELASTICITY I: FUNDAMENTALS	59
2.1. Introduction	59
2.2. Elastic Energy Density of an NLC in Terms of $n_{i,j} = \partial n_i / \partial x_j$	61
2.3. Electric Contribution to the Elastic Energy Density in Terms of $n_{i,j}$	65
2.4. Elastic Energy Density of NLC in Terms of $n_{i,j}$ and of $n_{i,jk}$	68
2.5. Surface-Like Contribution to the Elastic Energy Density: Well- and Ill-Posed Variational Problems	73
2.6. Saddle-Splay Term for One-Dimensional Deformation	77
2.7. Elastic Energy Density for NLC in Terms of Q_{ij}	78
2.8. Elastic Energy Density for NLC in the Second Order in $Q_{ij,k}$	86
2.9. Quadrupolar Properties Induced by Mechanical Deformations in NLC	90
2.10. Temperature Dependence of Surface Angles: Approximation for Constant S	99
2.11. Temperature Dependence of Surface Angles: Influence of the Spatial Variation of S	107
2.12. Ions and Anchoring Energy	114
3. THEORY OF ELASTICITY II: APPLICATIONS	121
3.1. Introduction	121
3.2. Hybrid Aligned NLC Cell: Strong Anchoring	124
3.3. Pretilted NLC Cell: Weak Anchoring	126
3.4. Hybrid Aligned NLC Cell: Weak Anchoring	127
3.5. Optical Path Difference Due to an NLC	132
3.6. Interaction with External Fields	135
3.6.1. Flexoelectric and dielectric interaction	135
3.6.2. Fixed charge	137
3.6.3. Fixed voltage	139
3.6.4. Dielectric interaction	142
3.6.5. Magnetic interaction	144
3.7. Fréedericksz Transition: Strong Anchoring	144
3.8. Fréedericksz Transition: Weak Anchoring	149
3.9. Fréedericksz Transition: Saturation Field	153
3.10. Hybrid Cell Subjected to an Electric Field Normal to the Walls	156

3.11. Tilted NLC Submitted to an Oblique Magnetic Field	159
3.12. Tilted NLC Sample Submitted to an Oblique Magnetic Field: General Case	166
3.13. Compensated NLC Submitted to an Electric Field Perpendicular to the Walls	168
3.14. Compensated NLC: Flexoelectric Effect	174
3.15. Influence of the Flexoelectric Effect on the NLC Orientation in a Hybrid Cell	178
3.16. Homeotropic NLC Under the Action of External Field: Influence of the Flexoelectric Effect on the Phase Diagram . .	180
3.17. Effect of the Splay-Bend Elastic Constant on the Fréedericksz Transition	182
3.18. Mechanical Model for Temperature Surface Transitions	186
3.19. Solid-Friction and Critical State in Lyotropic NLC Submitted to a Magnetic Field	193
3.19.1. Freedericksz transition and surface critical state	196
3.19.2. Oblique magnetic field	199
3.20. Director Orientation in a Cholesteric Liquid Crystal: Homeotropically Oriented Sample	202
3.21. Director Orientation in a Cholesteric Liquid Crystal: Homeotropically Oriented Sample Submitted to an Electric Field Perpendicular to the Walls	203
3.22. Hybrid Cell Submitted to a Magnetic Field Perpendicular to the Initial Deformation Plane	205
3.23. Rotation of the Polarization Plane of a Linearly Polarized Light Beam Inciding on a Hybrid Cell Submitted to a Magnetic Field	210
3.24. Rotation of the Polarization Plane of a Linearly Polarized Light Beam Inciding on a Hybrid Cell Submitted to an Electric Field	213
3.25. Planar NLC Sample Submitted to Two Crossed, Electric and Magnetic, Fields	215
4. MOLECULAR MODELS	223
4.1. Introduction	223
4.1.1. Limits of the pseudomolecular model	228

4.2.	Maier–Saupe and Nehring–Saupe Interactions in the NLC Phase	230
4.2.1.	The Maier–Saupe interaction	233
4.2.2.	The Nehring–Saupe interaction	236
4.3.	Maier–Saupe Interaction and Bulk Elastic Properties	237
4.3.1.	Maier–Saupe: Spherical approximation	238
4.3.2.	Maier–Saupe: Ellipsoidal approximation	240
4.4.	Quadrupolar Interaction: Bulk Properties	244
4.5.	Elastic Properties of an NLC Limited by a Surface. Maier–Saupe Model, with Spherical Interaction Volume	250
4.6.	Elastic Properties of an NLC Limited by a Surface. Maier–Saupe Model, with Ellipsoidal Interaction Volume	260
4.6.1.	Surface elastic constants	267
4.6.2.	Conclusions	270
4.7.	Generic Two-Body Interaction	271
4.8.	The First-Order Term in the Free Energy Density	278
4.9.	The Free Energy Density of the Reference State	279
4.10.	From the Nonlocal to the Elastic Description of NLC	279
4.11.	Bulk Elastic Properties of Magnetic Suspensions in Lyotropic NLC	286
5.	SUBSURFACE DEFORMATIONS IN NEMATICS	293
5.1.	Introduction	293
5.2.	Physical Meaning of Discontinuous Solutions	296
5.3.	K_{13} and the NLC Orientation	304
5.4.	K_{13} and the NLC Orientation: Ritz Method	311
5.5.	Effect of a Surface Field on the NLC Orientation	315
5.6.	Subsurface Deformation: Lattice Model	321
5.7.	Subsurface Deformation: Discrete Model	326
5.8.	Subsurface Deformation: Continuum Model	330
5.9.	Spatial Variation of the Elastic Constants and the Surface Energy	339
5.10.	Spatial Variation of the Elastic Constants and the Surface Energy: Antisymmetric Arrangement	347
5.11.	Spatial Variation of the Elastic Constants and the Surface Energy with Finite Anchoring Strengths	348
5.12.	Fréedericksz Transition with $K = K(z)$	351

5.13. Nonlocal Description of NLC	355
5.13.1. Maier–Saupe interaction	358
5.13.2. Nehring–Saupe interaction	359
5.13.3. Superposition of the Maier–Saupe and Nehring–Saupe interactions	361
5.13.4. Interaction with an external field	362
INDEX	365