are quasi-attractors. For three-dimensional systems, mathematically such attractors should contain infinitely stable periodic orbits, a finite number of which can only appear numerically due to the finite precision of computer experiments. They coexist with nontrivial hyperbolic sets. Such attractors are encountered in a lot of models, such as the Lorenz system, the *spiral-type and the double-scroll* attractor generated by a Chua's circuit, the Henon map, this for certain domains of the parameter space.

The complexity of quasi-attractor is essentially due to the existence of structurally unstable homoclinic orbits (homoclinic tangencies) not only in the system itself, but also in any system close to it. It results in a sensitivity of the attractor structure with respect to small variations of the parameters of the generating dynamical equation, i.e. *quasi attractors are structurally unstable.* Then such systems belong to Newhouse regions with the consequences given above.

In the n-dimensional case, $n > 3$, the situation becomes more complex and the first results (in particular a theorem showing that a system can be studied in a manifold of lower dimension) can be found in [Gonchenko *et al.*, 1993b, 1993c].

In addition to its interest in engineering applications, *Chua's circuit* generates a large number of complex fundamental dynamical phenomena. Indeed it is the source of different bifurcations giving rise to chaotic behaviors (period doubling cascade, breakdown of an invariant torus, etc.). The corresponding attractors are related to complex homoclinic heteroclinic structures. One of these attractors, the *double scroll*, characterized by the presence of three equilibrium points of saddle-focus type, arises from two nonsymmetric spiral attractors. It is different from other known attractors of autonomous three-dimensional systems in the sense that it is multistructural.

3. Conclusion

The important book by Madan [1993] collects many contributions devoted to applied and theoretical questions related to this circuit, which since this publication has given rise to many new developments. So the *synchronization of chaotic signals* generated by Chua's circuit leads to an increasing number of publications, with applications to secure communications [Lozi & Chua, 1993]. Moreover, a wide field of research is beginning to be opened through the use of a two- and three-dimensional grid of resistively coupled Chua's circuits. From such networks, waves and spatiotemporal chaos can be put in evidence with *travelling*, *spiral*, *target*, *scroll waves* [Chua & Pivka, 1995]. Here Chua's circuit is used as the basic cell in a discrete *cellular neural network* (*CNN*).

The study of *quasi-attractors* (which are generated in particular by Chua's circuit) does nothing but begin, and so gives a wide field of research. Such attractors cannot be made structurally stable via any finite parameter unfolding of the corresponding system. Arbitrarily small variations of parameters can lead to significant changes of the attractor structure. This results in the impossibility of attaining a complete description of their dynamics and their bifurcation space. Even for three-dimensional flows the results are not complete. A fortiori the extension to higher dimensional cases is a source of open problems for the future, because it is not trivial and provides the occasion to consider new dynamical phenomena [Shilnikov, 1994]. Chains of *Chua's* circuits may furnish a solution to such a matter. Nevertheless, a complete study of such processes being impossible, future research will only be concerned with some specific and typical properties of systems generating quasi-attractors. Related to the above question is the problem of the formulation of a *good model* [Gonchenko *et al.*, 1992], which has a sufficient number of parameters to analyze all possible bifurcations of the steady states, homoclinic and heteroclinic structures, etc. Applied aspects of quasi-attractors are mentioned in [Shilnikov, 1994]. They are concerned with the development of associative memories, and an approach for understanding the memory mechanisms. As indicated in Sec. 1 simple electronic oscillators originated many studies related to the qualitative theory of dynamical systems. It appears that Chua's circuit is now playing an equivalent role for the generation and understanding of complex dynamics, in relation with many applications.

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