

Chapter 2

The Turing Machine

The simplest “theoretical” digital computer is the Turing machine [44, 45]. Here the word “digital” indicates that the computer operates only with definite numbers (and does not use any quantum mechanical superposition of states). This machine was suggested by the British mathematician, A.M. Turing. The Turing machine has three parts, a tape divided into the squares, a scanner, and a dial, as in Fig. 2.1. This machine can write a symbol X or 1 in a blank square, and erase them. Any positive integer is written as a sequence of 1’s. For example, the number 5 corresponds to the sequence 11111. The symbol X indicates where a number begins or ends. For example, Fig. 2.1 shows two numbers 1 which are “prepared” for addition. The program for addition is presented in Tbl. 2.1. The symbol D is the command to “write the digit 1” in the corresponding square on the tape; X means “write X”; E means “erase”; R means “move the tape one square to the right”; L means “move tape one square to the left”. The numbers 1 to 6 after the letter indicate the command to “change the dial setting to this number”. The question mark represents a “mistake”; an exclamation mark means “job is completed”.

Now we shall describe the process of addition. First, the scanner sees the number 1 on the tape, and the dial setting 1. The instruction on the intersection (1,1) is R1: “move the tape one square to the right, and

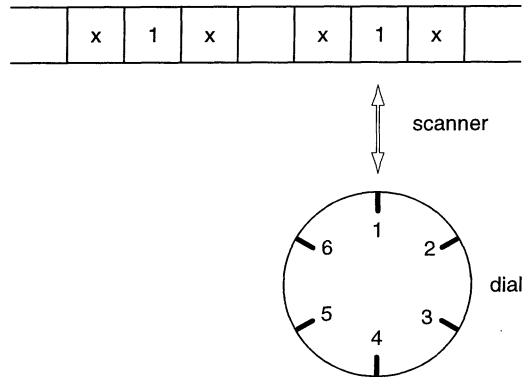


Figure 2.1: The Turing machine

dial setting

scanner symbol →

		X	1
1	D6	E2	R1
2	R2	E3	?
3	R3	E4	E5
4	L4	?	R6
5	L5	?	R1
6	X6	!	R3

Table 2.1: The program for addition in the Turing machine.

set the dial to 1". The second position is shown in Fig. 2.2. Here, the

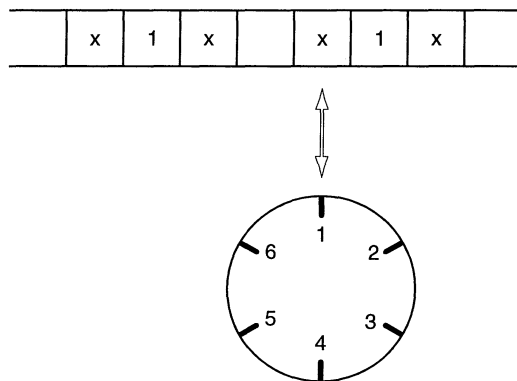


Figure 2.2: The second position of the Turing machine

scanner sees an X on the tape, and the dial setting is 1. The second instruction (1,X) is E2: "erase X, and set the dial to 2". The third position is shown in Fig. 2.3. The third instruction (2,□) is R2. Tbl. 2.2

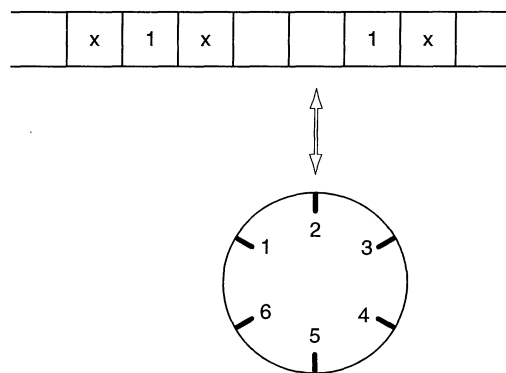



Figure 2.3: The third position of the Turing machine.

shows the sequence of positions and instructions following Fig. 2.3. The number in parentheses inside the square indicates the dial setting at the

instructions 

x	1	x	(2)		1	x	R2
x	1	x(2)			1	x	E3
x	1	(3)			1	x	R3
x	1(3)				1	x	E5
x	(5)				1	x	L5
x		(5)			1	x	L5
x			(5)		1	x	L5
x				(5)	1	x	L5
x					1(5)	x	R1
x				(1)	1	x	D6
x				1(6)	1	x	R3
x			(3)	1	1	x	R3
x		(3)		1	1	x	R3
x	(3)			1	1	x	R3
x(3)				1	1	x	E4
(4)				1	1	x	L4
	(4)			1	1	x	L4
		(4)		1	1	x	L4
			(4)	1	1	x	L4
				1(4)	1	x	R6
			(6)	1	1	x	X6
			x(6)	1	1	x	!

Table 2.2: The sequence of positions and instructions following Fig. 2.3.

position of the scanner. For example, 1(5) indicates that the scanner points to the square whose index is 1 and the dial setting is 5. If the scanner points the blank square, and the dial setting is 6, the corresponding notation in the Tbl. 2.2 is (6). The last row in the Tbl. 2.2 shows the result of addition: $1+1=2$. The program for multiplication requires 15 numbers on the dial, but the idea of the programming is the same.

The Turing machine has the same main components that any computer has. The writing and erasing elements represent the arithmetic unit, which perform calculations. The table of instructions (Tbl. 2.2) is the control unit. The tape and the dial are the memory unit.