

Preface

The present book is a translation and an expansion of an introductory text based on a lecture series delivered in Peking University (the People's Republic of China) in 1980 by a renowned leader in differential geometry, S.S. Chern. The original Chinese text resulted from the efforts of several colleagues^a, and, in its final form^b, was compiled by Wei-Huan Chen of Peking University. The final rendition of this English translation is carried out by the undersigned, under the guidance of S.S. Chern. It has been revised from a preliminary draft by Hung-Chieh Chang, which was prepared under the supervision of Professor T.T. Moh of Purdue University.

This translation aims at preserving, as far as possible, both the contents and style of Professor Chern's lectures, the hallmarks of which are simplicity, directness, and economy of approach together with in-depth treatments of fundamental topics. It should be suitable as a text or a work of reference for a wide audience, including (but not limited to) advanced undergraduate and beginning graduate students in mathematics, as well as physicists interested in the diverse applications of differential geometry to physics. Our hope is that the material in this text will provide a solid and comprehensive background for more advanced and specialized studies.

It is Professor Chern's opinion that the time is ripe for the subject of Finsler geometry to occupy a more prominent position within university curricula in basic differential geometry. It was already alluded to by Riemann in his famous Habilitation speech of 1854; and its relevance to the calculus of variations was stressed by Hilbert in his 1900 Paris Lecture. Since Finsler's thesis work on it in 1918, the subject has seen many important developments, but has lacked the kind of coherence that characterizes Riemannian geometry. Some remarkable recent work has shown, however, that the more natural starting point of Riemannian geometry is the more general Finsler setting, and that many of the beautiful and deep results in the former have Finslerian counterparts. Professor Chern himself, beginning with his early work in the 1940's and in recent collaborations with David Bao, has initiated crucial steps and paved the way in this research.

In view of these developments, a new and rather lengthy chapter prepared by Kai S. Lam and S.S. Chern on Finsler geometry (Chapter Eight) has been added. The last section in Chapter Five of the original Chinese text on completeness in Riemannian geometry has also been revised and reincorporated as section 7 of the new chapter, which treats completeness in Finsler geometry.

To bring the entire subject of differential geometry into perspective, Professor Chern has written a valuable piece, "Historical Notes", specially for the

present English edition. This appears as the new Appendix A. Appendix B, entitled "Differential Geometry and Theoretical Physics", was originally authored by Professor Chern in Chinese and included in the Chinese text. The English translation, by the undersigned, appears for the first time in this book. It is hoped that this essay will stimulate a degree of fruitful discussion between mathematicians and physicists.

Professor Chern is well known for his masterful synthesis of deep geometrical insights and skillful calculations. The present text will bear witness to this immensely fruitful mathematical style. A central theme of the text is that global and local problems of differential geometry are equally interesting and important. Even though local objects such as coordinates in a manifold are devoid of intrinsic meaning, local tools, such as Cartan's exterior differential calculus and Ricci's tensor analysis, are extremely useful in the study of manifolds. Hence these tools have been developed and used extensively; but at the same time, the importance of intrinsic objects with invariant properties under a change of coordinates, such as tangent vector fields, differential forms, etc., is also stressed. Throughout the text, the relationship between local and global properties of a manifold, as exemplified, for example, by the Gauss-Bonnet Theorem (Chapter Five) and the Chern classes (Chapter Seven), is emphasized.

As a physicist with relatively little formal training in mathematics, I take great pleasure in expressing my sincere gratitude to Professor Chern. Not only has he graciously put up with the plodding attempts of a novice, but has also, over the course of many months, provided me with generous support and guidance. In addition, he took great pains to introduce me to the beautiful and fascinating developments in Finsler geometry. I have deeply benefitted from this unique opportunity to collaborate with and learn from a great mathematician. The new chapter on Finsler geometry has relied heavily on the joint work of Bao and Chern [Houston Journal of Mathematics, Vol. 19, No. 1, 135-180 (1993)], and preliminary drafts of an upcoming comprehensive treatise, "An Introduction to Riemann-Finsler Geometry," by Bao, Chern and Shen, to be published by Springer-Verlag. I am deeply grateful to Professor David Bao of the University of Houston, to Professor Zhongmin Shen of Indiana University-Purdue University at Indianapolis, and again to Professor Chern, for allowing me to draw from these materials. A special note of heartfelt thanks is owed to David Bao, who has rendered inestimable help in the preparation of the new chapter, by carefully going over the drafts, offering freely expert advice, and generously providing much needed reference material. In many ways, our chapter on Finsler geometry may be viewed as providing an introduction to the Bao-Chern-Shen treatise mentioned above, and the serious reader who wishes to explore the subject at greater depth is well-advised to pursue that definitive work.

I would also like to thank my colleagues in both the Physics and Mathematics Departments of my home institution for their various kind acts of assistance, support, and encouragement, especially physicists John Fang and Soumya Chakravarti, mathematical physicist Martin Nakashima, and mathematicians Bernard Banks and Charles Amelin. To physicist Dr. Barbara Hoeling I owe a special word of gratitude for helping me translate an early paper by L. Berwald from German to English. In addition, I am grateful to the Faculty Sabbatical Program of the California State University for providing the necessary freedom and time for this project. Last but not least, I wish to thank my wife, Dr. Bonnie J. Buratti, and our three boys, Nathan, Reuben, and Aaron, for their simply being part of a wonderful and supportive family.

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Kai S. Lam
California State Polytechnic University, Pomona

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^b*Lectures on Differential Geometry* (微分几何讲义) by S.S. Chern and Wei-Huan Chen, Peking University Press, 1983. The present translation is based on this edition. A second Chinese edition, under the same title and with no textual changes from the first, was published in 1990 by the Lien Ching Ch' u Pan Shih Yeh Co., Taipei, Republic of China.