

# Preface

This book is based on lecture notes for the introductory course on modern, coordinate-free differential geometry which is taken by our first-year theoretical physics PhD students, or by students attending the one-year MSc course “Fundamental Fields and Forces” at Imperial College.

The course is concerned entirely with the mathematics itself, although the emphasis and detailed topics have been chosen with an eye on the way in which differential geometry is applied to theoretical physics these days. Such applications include not only the traditional area of general relativity, but also the theory of Yang-Mills fields, non-linear sigma models, superstring theory, and other types of non-linear field systems that feature in modern elementary particle theory and quantum gravity.

The course is in four parts dealing with, respectively, (i) an introduction to general topology; (ii) introductory coordinate-free differential geometry; (iii) geometrical aspects of the theory of Lie groups and Lie group actions on manifolds; and (iv) the basic ideas of fibre bundle theory.

The first chapter contains a short introduction to general topology with the aim of providing the necessary prerequisites for the later chapters on differential geometry and fibre bundle theory. The treatment is a little idiosyncratic in so far as I wanted to emphasise certain algebraic aspects of topology that are not normally mentioned in introductory mathematics texts but which are of potential interest and importance in the use of topology in theoretical physics.

The second and third chapters contain an introduction to differential geometry proper. In preparing this part of the text, I was particularly conscious of the difficulty which physics graduate students often experience when being exposed for the first time to the rather abstract ideas of differential geometry. In particular, I have laid considerable stress on the basic ideas of 'tangent space structure', which I develop from several different points of view: some geometric, some more algebraic. My experience in teaching this subject for a number of years is that a firm understanding of the various ways of describing tangent spaces is the key to attaining a grasp of differential geometry that goes beyond just a superficial acquiescence in the jargon of the subject. I have not included any material on Riemannian geometry as this aspect of the subject is well covered in many existing texts on differential geometry and/or general relativity.

Chapter four is concerned with the theory of Lie groups, and the action of Lie groups on differentiable manifolds. I have tried here to emphasise the geometrical foundations of the connection between Lie groups and Lie algebras, but the latter subject is not treated in any detail and readers not familiar with this topic should supplement the text at this point.

The theory of fibre bundles is introduced in chapter five, with a treatment that emphasises the theory of principle bundles and their associated bundles. The final chapter contains an introduction to the theory of connections and their use in Yang-Mills theory. This is fairly brief since many excellent introductions to the subject aimed at physicists have been published in recent years, and there is no great point in replicating that material in detail here.

The second edition of this book differs from the first mainly by the addition of the chapter on general topology; it has also been completely reset in LaTeX, thus allowing for a more extensive index. In addition, I have taken the opportunity to correct misprints in the original text, and I have included a few more worked examples. A number of short explanatory remarks have been added in places where readers and students have suggested that it might be helpful: I am most grateful to all those who drew my attention to such deficiencies in the original text. However, I have resisted the attention to

add substantial amounts of new material—other than the chapter on topology—since I wanted to retain the flavour of the original as bona fide lecture *notes* that could reasonably be read in their entirety by a student who sought an overall introduction to the subject.

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