

1	An Introduction to Topology	1
1.1	Preliminary Remarks	1
1.1.1	Remarks on differential geometry	1
1.1.2	Remarks on topology	2
1.2	Metric Spaces	3
1.2.1	The simple idea of convergence	3
1.2.2	The idea of a metric space	5
1.2.3	Examples of metric spaces	8
1.2.4	Operations on metrics	10
1.2.5	Some topological concepts in metric spaces	11
1.3	Partially Ordered Sets and Lattices	14
1.3.1	Partially ordered sets	14
1.3.2	Lattices	18
1.4	General Topology	23
1.4.1	An example of non-metric convergence	23
1.4.2	The idea of a neighbourhood space	25
1.4.3	Topological spaces	32
1.4.4	Some examples of topologies on a finite set	37
1.4.5	A topology as a lattice	40
1.4.6	The lattice of topologies $\tau(X)$ on a set X	42

1.4.7	Some properties of convergence in a general topological space	45
1.4.8	The idea of a compact space	46
1.4.9	Maps between topological spaces	48
1.4.10	The idea of a homeomorphism	51
1.4.11	Separation axioms	52
1.4.12	Frames and locales	54
2	Differentiable Manifolds	59
2.1	Preliminary Remarks	59
2.2	The Main Definitions	60
2.2.1	Coordinate charts	60
2.2.2	Some examples of differentiable manifolds . . .	64
2.2.3	Differentiable maps	68
2.3	Tangent Spaces	70
2.3.1	The intuitive idea	70
2.3.2	A tangent vector as an equivalence class of curves	72
2.3.3	The vector space structure on $T_p\mathcal{M}$	76
2.3.4	The push-forward of an equivalence class of curves.	77
2.3.5	Tangent vectors as derivations	79
2.3.6	The tangent space T_vV of a vector space V . . .	90
2.3.7	A simple example of the push-forward operation	91
2.3.8	The tangent space of a product manifold	92
3	Vector Fields and n-Forms	97
3.1	Vector Fields	97
3.1.1	The main definition	97
3.1.2	The vector field commutator	102
3.1.3	h -related vector fields	104

3.2	Integral Curves and Flows	107
3.2.1	Complete vector fields	107
3.2.2	One-parameter groups of diffeomorphisms	111
3.2.3	Local flows	115
3.2.4	Some concrete examples of integral curves and flows	117
3.3	Cotangent Vectors	121
3.3.1	The algebraic dual of a vector space	121
3.3.2	The main definitions	123
3.3.3	The pull-back of a one-form	126
3.3.4	A simple example of the pull-back operation	129
3.3.5	The Lie derivative	130
3.4	General Tensors and n -Forms	132
3.4.1	The tensor product operation	132
3.4.2	The idea of an n -form	135
3.4.3	The definition of the exterior derivative	137
3.4.4	The local nature of the exterior derivative	138
3.5	DeRham Cohomology	140
4	Lie Groups	149
4.1	The Basic Ideas	149
4.1.1	The first definitions	149
4.1.2	The orthogonal group	155
4.2	The Lie Algebra of a Lie Group	157
4.2.1	Left-invariant vector fields	157
4.2.2	The completeness of a left-invariant vector field	162
4.2.3	The exponential map	165
4.2.4	The Lie algebra of $GL(n, \mathbb{R})$	169
4.3	Left-Invariant Forms	170

4.3.1	The basic definitions	170
4.3.2	The Cartan-Maurer form	172
4.4	Transformation Groups	175
4.4.1	The basic definitions	175
4.4.2	Different types of group action	179
4.4.3	The main theorem for transitive group actions	183
4.4.4	Some important transitive actions	185
4.5	Infinitesimal Transformations	190
4.5.1	The induced vector field	190
4.5.2	The main result	195
5	Fibre Bundles	199
5.1	Bundles in General	199
5.1.1	Introduction	199
5.1.2	The definition of a bundle	201
5.1.3	The idea of a cross-section	207
5.1.4	Covering spaces and sheaves	210
5.1.5	The definition of a sub-bundle	213
5.1.6	Maps between bundles	214
5.1.7	The pull-back operation	216
5.1.8	Universal bundles	218
5.2	Principal Fibre Bundles	220
5.2.1	The main definition	220
5.2.2	Principal bundle maps	224
5.2.3	Cross-sections of a principal bundle	230
5.3	Associated Bundles	232
5.3.1	The main definition	232
5.3.2	Associated bundle maps	236

<i>CONTENTS</i>	xiii
5.3.3 Restricting and extending the structure group	240
5.3.4 Riemannian metrics as reductions of $\mathbf{B}(\mathcal{M})$	243
5.3.5 Cross-sections as functions on the principle bundle	246
5.4 Vector Bundles	248
5.4.1 The main definitions	248
5.4.2 Vector bundles as associated bundles	249
6 Connections in a Bundle	253
6.1 Connections in a Principal Bundle	253
6.1.1 The definition of a connection	253
6.1.2 Local representatives of a connection	256
6.1.3 Local gauge transformations	258
6.1.4 Connections in the frame bundle	261
6.2 Parallel Transport	262
6.2.1 Parallel transport in a principal bundle	262
6.2.2 Parallel transport in an associated bundle	267
6.2.3 Covariant differentiation	269
6.2.4 The curvature two-form	271
BIBLIOGRAPHY	277
INDEX	281