Preface

GROUPS, MATHEMATICS, PHYSICS, ART

Group theory is a diverse, extensive subject, playing roles, often fundamental, in numerous areas — there are many different ways in which it is useful, enlightening, often necessary. This book considers thus only a small part of a broad, substantial topic. While a paramount goal is, of course, teaching the theory, certainly providing the basis for application, its deeper rationale is development of (part of) the foundation for the study of group theory more generally, and of mathematics, physics, chemistry, crystallography — and beyond. The hope is to teach group theory, explaining how it is used, not merely that part that it covers — and to nurture the skills to develop and apply it, helping and stimulating the reader to gain thorough understanding.

Group theory is important, essential, for students in many different fields, for many different types of researchers. The book tries to achieve the impossible by appealing to all. It can be used by those only slightly interested, to get a quick definition, a brief introduction to a concept, a slight understanding of what is being read elsewhere, even of only vocabulary. And it can, hopefully, be studied for deeper and deeper understanding, as profound as wished. For those who really study it, who make serious attempts at many of the problems, who think about questions raised, it is designed to develop not only insightful, profound knowledge of groups, and applications, but more generally of mathematics, physics, chemistry, even philosophy. For such readers the desire is to aid also in the development of those skills and habits that lead to success more generally.

How difficult is this book? It is written to be easy to read and understand, for those who want an introduction to the subject, a reading knowledge, fundamental concepts, vocabulary. And it should not be more difficult for readers who wish to go deeper, although of course that requires more work. But it also attempts to provide the material, the questions, the problems, the challenge, the irritation, the skepti-

cism, so that those who wish a very deep understanding, who wish potent skills, and not only in this subject, will have the stimulation and exercises to help obtain them.

Crystals and molecules play a preeminent role in physics and chemistry (and technology and economics), and their groups of transformations are basic in determining their properties and those of objects that compose them. But beyond such applications, they provide subjects quite suited for the development and illustration of many general concepts of group theory, geometry, mathematics. It is thus surprising that there are have been so few books devoted to the theory of these groups. There have been (in the past) books available, but there seems to never have been a comprehensive treatment of this quite vast subject. This book is certainly not that, and perhaps such is impossible. But it attempts to at least introduce the major parts of the subject, to show how they are related, and what their importance is — and to provide guidance (certainly references) to those who wish to learn more, to go deeper.

Because of the extent of the subject it is impossible to cover more than a small part. In many cases material is merely mentioned, often in problems, to inform the reader that it exists, to at least provide definitions, and also to indicate where further information can be found. Thus, of regrettable necessity, many topics had to be only outlined, but with references to other discussions, perhaps in more depth. And if the reader should find all these discussions are also only outlines, the conclusions to be drawn would be obvious. Often a topic is referred to only to be able to provide references. Unfortunately these are in too many cases to books no longer in print, frequently long out-of-print. Yet the material that they contain, even just the approach, is often not found anywhere else. As they disappear, the knowledge within them, typically gained with so much difficulty and so useful, disappears also. Thus one hope is to at least keep alive the memory of books long gone, but whose disappearance is a substantial loss. A book at least differs from a person — it can be brought back from the dead. Perhaps if memory can be kept alive, some books themselves might be restored to life.

This book is, of course, aimed at providing knowledge of group theory, to enable its readers (to apply and develop it), specifically the part that deals with discrete, particularly crystallographic, groups, and to use these applications to help in mastering groups, and the systems to which they can be applied. But more, what is emphasized is understanding, intuition, not only about groups, crystals, molecules, but about mathematics, physics, chemistry, crystallography, and related fields, and about understanding. It is designed to push the reader to

understand the theory, the reasons for it, for the properties of (these) groups, for what we have decided should be the properties, and why. So it aims to stimulate, to goad, the reader to think about, not only the material, but its foundations, its reasons, about what is being read. Those in education know that too often we do not educate our students, we program them. Like a good computer program they are able to deal well with material for which they have been programmed. But too often, just as a computer program, they do not understand what they are doing, or why. They do not understand the reasons the particular software has been read into them. They are unable to think about what they are doing, to extend their capabilities to areas other than those for which they have been programmed. Obviously researchers should be able to deal with novelty, but too often they are able to merely apply routinely their built-in software to subjects at most slightly different then those taught to them. And educators also must be able to think innovatively. How else can they teach their students to do so? This is part of the underlying philosophy of the book and the way it was written (like the first book of this series [Mirman (1995a)]). Is the approach correct? Can it achieve what it tries to? Can any approach? This is an experiment. Hopefully it will not only teach the material, and its understanding, to enable readers to deal with it, but also it will teach those who educate a little about how, and about how not, to do so — or at least stimulate (or disturb) them enough so that they will consider these questions. A fundamental rationale is to raise such issues, to prod the reader to think — and to provoke those who use and teach from this book, to also think, not only about group theory, mathematics, physics, but about education. It aims to stimulate others to try to devise ways of better educating, their students and themselves, and hopefully to spread what they have learned, so others may learn, may be sensitized, aroused, so that they too will look — and find — ways of better, not merely programing, but educating.

What is learned in (good) formal education is never sufficient for a successful career. Rather it provides the foundation for necessary further learning. A successful education imparts not only knowledge — necessary for the acquisition of further knowledge — but skills, to go deeper into subjects, to utilize and extend knowledge and skills, to gain further knowledge. It provides not only depth, and the experience to penetrate further, but breadth, flexibility, abilities to go into other fields, to deal with other forms of knowledge, to acquire and use them; it provides the skills necessary to learn other skills.

This is a major aspect of the philosophy of these books. Not only to teach the subjects and the skills to use them, but also those needed to learn more, about these fields, about others, and the capacity to

deal with whatever problems the reader encounters, in whatever fields, however they may arise. For success, especially during uncertainty, breadth and adaptability are essential.

One skill needed by a researcher is judgment — what problems should be attempted? Many problems here are partially trivial, partially research, partially impossible. Deciding what can, and what cannot be done, what is worth trying, is valuable exercise.

Although the question of rigor has been discussed previously [Mirman (1995a), sec. P.5, p. xv], it is worth emphasizing again, for this also is part of the underlying philosophy, and here quite relevant. An exposition is rigorous if it is written in a way so that it can be widely, and easily, understood, thus easily checked. Having been checked by many people, we can be more confident that there are no errors, that all assumptions have been stated. By this standard much of present mathematics and physics is highly nonrigorous. Obscurantism is not rigor, despite the much too popular belief. It is just the opposite.

What is so disturbing is not merely the way much material is so often presented, or written so as not to be presented, but that there is no attempt to explain or understand it. Science becomes a religion; these are the accepted beliefs, there are no reasons for them, there is no need to explain them. Read not only many, but (at least, close to) all, books on these topics. Consider the number of subjects presented, not discussed, without explanation, without justification, without reasons, without understanding, without even the hint of why they are true (if they are), where they came from, or where further discussion or justification can be found (for they cannot). This is not education, it does not lead to understanding. Here then we attempt to fill in (unfortunately, necessarily only some of) these gaps, or at least point them out — and to emphasize how often books on mathematics and science present not explanations, rationales, proofs, but merely beliefs. And very often what everyone believes, while it may be the truth, is not the whole truth. Someone learning the subject should understand this, should know what is true, what is not true, what is not fully true.

For the insightful comprehension we hope the reader will gain, it is important, even essential, to have different views of every (or at least the most important) aspects, and to see how they are related. Thus there are topics covered in great depth. This also allows flexibility. The reader, or the instructor, has available much material from which the choice most fitting for a particular case can be made. There are fewer restrictions because of choices the writer has made.

And to achieve understanding it is necessary to stimulate the reader to think, so to raise questions, doubts even paradoxes, mysteries, contradictions. This underlies many of problems.

While an attempt has been made to explain and justify much material very fully, it is impossible to do this for everything. Much had to be just outlined, with the material taken from standard sources (which are referenced so the reader can find, hopefully, more comprehensive discussions), and many of these took their material from other standard sources. No claim is made that all material taken from such sources has been carefully checked, and perhaps the standard sources did not check everything taken from previous standard sources. It is not unheard-of for errors to be propagated because "facts" become well-known and established, so never checked. Readers should always be very careful. Teaching this is another hope.

Attitudes of researchers, physicists, mathematicians — any researchers — are essential, for success and the production of useful and meaningful work. Of great importance is open-minded skepticism, being willing to examine with care and understanding new ideas, approaches, results no matter how disturbing or contrary to previous beliefs, but also care and skepticism, not only with these but also with accepted beliefs, establishment views. Too many people have instead a closed-minded credulity, closed to anything different from what they are told to believe, but credulous for the sanctioned, the authorized, no matter weakly supported, even absurd, that may be. Too often we accept, uncritically, what we are told, what is on a piece of paper. So the book tries to instill an attitude of open-minded skepticism, to break the connection between appearance of a statement on paper or in a book and acceptance of it. Of course, only slight progress can be made, but it is hoped that at least this is possible — and that others will be stimulated to also try to develop, the so essential, open-minded skepticism. Thus (perhaps too) occasionally, doubts are raised about the statements being presented so to accustom the reader to skepticism, to care, to the habit of checking. Open-mindedness is harder, but perhaps once thought becomes a habit, useful doubt also will.

The philosophy underlying this book is the same as the first one [Mirman (1995a)]. To help the reader understand what is being attempted, an explanation of the approach is attached.

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