

Chapter 1

INTRODUCTION

Linear programming (LP) is formulated with a single criterion (objective) and a single (fixed) resource availability level (right hand side). For the past five decades, linear programming has been widely applied in many real-world decision-making problems. However, like any great advancement, linear programming is not a perfect tool. For example, if a decision problem involves multiple conflicting criteria, such as maximizing profit while minimizing production cost, linear programming may have limitations in effectively addressing possible tradeoffs among the criteria. This shortcoming has been overcome by mathematical models known as multiple criteria (MC) linear programming. MC linear programming improves the value of linear programming by changing its single criterion to multiple criteria.

Even though the response to MC linear programming from practitioners is very positive, MC linear programming is not the best modeling tool. For instance, if an MC model is used to identify a best decision (solution) for investing ten million dollars in a stock market, its right hand side level (resource availability) should not be fixed. The level of which the future stock share value is going up should differ from that of which the future stock share value is going down. Can the linear programming framework be made more flexible and realistic to reflect the real world? Multiple criteria and multiple constraint-level (MC²) linear programming is an answer. In addition to the extension of MC linear programming from linear programming, MC² linear programming explicitly expresses multiple (discrete) levels of resource availability. In other words, a single fixed feasible set of linear programming is replaced by several flexible feasible sets.

In this chapter, we first review the history of linear programming, MC linear programming, and MC² linear programming in Section 1.1. Then, to motivate the reader's interest, we use a simple example to illustrate the concepts of linear programming, MC linear programming, and MC² linear programming, respectively in Sections 1.2, 1.3 and 1.4. Finally, we give some remarks for the further reading in Section 1.5.

1.1 Brief History

1. The original invention of linear programming can be traced back to the late 1930's when Russian mathematician L. K. Kantorovich found that a class of linear programming models could be used for production planning. In the United States, the duality theorem which serves as a mathematical foundation of linear programming was initiated by J. von Neumann in 1944. In 1947, G. B. Dantzig formulated the first linear program for the U.S. airforce's planning process. This formulation had its origins in the economic input-output model developed by W. Leontief and his associates in 1941 and 1945. In order to solve linear programming problems, G. B. Dantzig created the well-known "simplex method", which was first tested by a diet problem based on the home economist George Stigler's data. In 1948, T.C. Koopmans and G. B. Dantzig named this new linear structure "linear programming." Later, T.C. Koopmans applied linear programming as a framework to analyze a class of economic phenomena. In the meantime, S. A. Charnes and W. W. Cooper made their fundamental contributions to model a number of business problems through the techniques of Linear Programming. Since the 1950's, linear programming has been often applied to formulate and solve various resource allocation, scheduling, distribution, oil refinement, and transportation problems. This also had a great influence on the development of computers. In 1975, the Royal Sweden Academy of Sciences awarded a Nobel Prize in economics to both L. K. Kantorovich and T.C. Koopmans.

After the initiation of the simplex method, researchers have been proposing numerous algorithms to reduce the computation complexity in solving large-scale linear programming problems. Among them, two algorithms are particularly significant. The first is the "ellipsoid method" proved by L. G. Khachian in 1979. This method theoretically shows a polynomial-time bound to find an optimal solution of a linear program, which outperforms the simplex method that uses an exponential time to find an optimal solution. However, the ellipsoid method has not been applied to the real-world applications. The second is the "interior point method" developed by N. Karmarkar in 1984. Compared to the simplex method that searches the optimal solution from the corner points along the edges of the feasible region, the interior point method finds the optimal solution through the interior points of the feasible region. Some reports have recently shown that this approach seems to be faster than the simplex method in locating an optimal

solution for a number of large-scale practical linear programming problems. However, the augments of relative superiority of these methods both practically and theoretically are still going on.

As early as 1951, D. Gale, H.W. Kuhn and A.W. Tucker noticed that the basic structure of linear programming can be mathematically extended into "a linear matrix programming problem", which since 1970's has been well known as "MC linear programming." In linear matrix programming, the single objective coefficients (a vector) of linear programming are replaced by several objective coefficients (a matrix). This extension opened a new technique to study the tradeoffs of conflicting criteria in real-world problems. In 1974, M. Zeleny published the first MC linear programming book based on his Ph.D. thesis under guidance and advice of P. L. Yu. T. Gal, in his 1979's book, extended simplex methods to a series of parametric programming. The field of MC linear programming in both theory and applications has been booming for the last three decades. During the period of time, the International Society on Multiple Criteria Decision Making (MCDM) was established to exchange ideas, scholastic works, and application experiences among researchers and practitioners throughout the world. By 2000, this organization has held 15 international conferences, published a newsletter twice each year, and has about 1,300 members from 82 countries.

In 1979, P. L. Yu and L. Seiford observed that because the objective coefficients of a primal linear program is the constraint level of its dual program (by the duality theory), the multiple constraint levels can be built in the structure of linear system like multiple criteria. This led to the concept, formulation, and development of MC^2 linear programming. The author of this book began to work on this extended technique in 1985 when he joined the Ph.D. program at the University of Kansas as a graduate student and assistant of P. L. Yu. Part of motivation of the author's dissertation was from Y. R. Lee (1986). Much of the theories and many methods of this book are based on the work with P. L. Yu. In the last decade, the author, as the principle investigator and a group of professors and graduate students have been systematically working on theory and applications of MC^2 linear programming problems. Members of this group are from twelve universities in USA, China, Korea, Germany, and Hungary. The main research results of the group range from theoretical bases to broad areas of real-world applications. The theoretical bases include: relationships between linear programming, multiple criteria linear programming, and MC^2 linear programming; integer MC^2 linear programming and MC^2

transportation model; fuzzy MC² linear programming problems and fuzzy duality theory in MC² linear programming; optimal system designs and contingency plans; decision support system of solving complex system design problems; and MC² computer software development. The application areas include accounting, agricultural policy, finance, management information systems, production planning, and telecommunications management.

1.2 The Linear Programming Problem

The Trend Computing Company produces the variety of PC's and Laptops. Recently, the R & D department proposes two types of the new generation PC. Each type consists of hardware (monitor, hard drive, keyboard, etc.) and software (Windows, Internet Browser, Lotus Notes, etc.). Suppose that 3 units of hardware and 8 units of software are needed to assemble a type 1 PC, while 2 units of hardware and 3 units of software are used to make a type 2 PC. Each type 1 will produce \$400 as profit and each type 2's profit is \$200. If the *production manager* believes that the company can produce 40,000 units of hardware and license 90,000 units of software, then how many PC's of each type should the company assemble to achieve the maximal total profit?

Table 1.1 The PC Problem for LP

Resource	Type 1	Type 2	Availability Level (units)
Hardware	3	2	40,000
Software	8	3	90,000
Profit (\$)	400	200	

Table 1.1 summarizes the information given in the above PC problem. In order to find the best production mix for this problem, we first define the following **decision variables**:

Let x_1 = the number of type 1 PC made, and
 x_2 = the number of type 2 PC made.

Then, the **objective (criterion) function** that shows the maximal total profit can be expressed as

$$\text{Max } 400 x_1 + 200 x_2$$

The **constraints** of hardware and software for the production mix can be represented by:

$$3 x_1 + 2x_2 \leq 40,000 \quad (\text{Hardware})$$

$$8 x_1 + 3x_2 \leq 90,000 \quad (\text{Software}).$$

Since both variables x_1 and x_2 are the quantities and cannot be negative, the **nonnegativity conditions** x_1 and $x_2 \geq 0$ should be considered.

Finally, a **linear program** of the PC problem (M1.1) is formulated as

$$\begin{array}{lll} \text{Max} & 400 x_1 + 200 x_2 & (\text{Profit}) \\ \text{Subject to} & 3 x_1 + 2x_2 \leq 40,000 & (\text{Hardware}) \\ & 8 x_1 + 3x_2 \leq 90,000 & (\text{Software}) \\ & x_1, x_2 \geq 0 & \end{array}$$

Using the **matrix representation**, the formulation is

$$\begin{array}{l} \text{Max } (400 \quad 200) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\ \text{Subject to } \begin{pmatrix} 3 & 2 \\ 8 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 40,000 \\ 90,000 \end{pmatrix} \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \geq 0 \end{array}$$

In this representation,

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

is called **decision vector**;

$$(400 \quad 200)$$

is the **criterion coefficients**;

$$\begin{pmatrix} 3 & 2 \\ 8 & 3 \end{pmatrix}$$

is the **consumption matrix**; and

$$\begin{pmatrix} 40,000 \\ 90,000 \end{pmatrix}$$

is the **availability vector**.

Methods of how to identify the optimal solution for this linear programming problem will be discussed in Chapter 2. Note that the linear programming problem for the PC production has only single criterion (maximizing the total profit) and single resource availability level (from the production manager's point of view). However, in real-world situations, the company may want to simultaneously achieve the maximal production quantities and the maximal profit. The production quantities differ from the profit. Therefore, if these two criteria are under consideration, the linear program has to be extended and reformed. This leads us to think about an MC linear program for the PC problem in the following subsection.

1.3 The MC Linear Programming Problem

In order to see the significance of the extension of linear programming through the PC production problem, we first revise Table 1.1 to reflect the second criterion: the maximal production quantities. Thus, the new Table 1.2 has one more row of data for production quantities than Table 1.1.

By adding the second objective function

$$\text{Max} \quad x_1 + x_2 \qquad \qquad \qquad \text{(Production quantities)}$$

to the LP formulation in Section 1.2, the **MC (or two criteria) linear program** (M1.2) for the PC problem can be expressed as

Table 1.2 The PC Problem for MC

Resource	Type 1	Type 2	Availability (units)	Level
Hardware	3	2	40,000	
Software	8	3	90,000	
Profit (\$)	400	200		
Production Quantities	1	1		

$$\begin{array}{lll}
 \text{Max} & 400 x_1 + 200 x_2 & \text{(Profit)} \\
 \text{Max} & x_1 + x_2 & \text{(Production quantities)} \\
 \text{Subject to} & 3 x_1 + 2x_2 \leq 40,000 & \text{(Hardware)} \\
 & 8 x_1 + 3x_2 \leq 90,000 & \text{(Software)} \\
 & x_1, x_2 \geq 0 &
 \end{array}$$

Mathematically speaking, finding the ideal solution that simultaneously represents the maximal production quantities and the maximal profit is almost impossible. However, the theory of MC linear programming allows us to study the tradeoffs of the **criteria space**. In this case, the criteria space is a two dimensional plane consisting of production quantities and profit. The “optimal” solution of MC linear programming is interpreted as an **efficient** or **nondominated solution** that is the acceptable compromise or tradeoff between the multiple criteria. To conduct the tradeoff analysis of multiple criteria, we often put a **weight parameter** on each criterion under consideration. The nondominated solution therefore is a function of weight parameters. Whenever the value of its weights changes, the corresponding nondominated solution may vary. In practice, if the set of nondominated solutions is identified, the decision maker may choose the final solution from one of nondominated solutions for implementation. If the set of nondominated solutions cannot be identified or is difficult to be found, the decision maker may interact with the analyst for identifying a preferred or “near-optimal” solution.

In the PC problem, let λ_1 be the weight parameter for the profit and λ_2 be the weight parameter for production quantities. The MC program of the PC problem (M1.2) becomes a parametric linear program:

$$\begin{array}{lll}
 \text{Max} & \lambda_1 (400 x_1 + 200 x_2) + \lambda_2 (x_1 + x_2) & \\
 \text{Subject to} & 3 x_1 + 2x_2 \leq 40,000 & \text{(Hardware)} \\
 & 8 x_1 + 3x_2 \leq 90,000 & \text{(Software)} \\
 & x_1, x_2 \geq 0 &
 \end{array}$$

The matrix form of this problem is

$$\begin{aligned} & \text{Max} \begin{pmatrix} \lambda_1 & \lambda_2 \end{pmatrix} \begin{pmatrix} 400 & 200 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\ & \text{Subject to} \begin{pmatrix} 3 & 2 \\ 8 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 40,000 \\ 90,000 \end{pmatrix} \\ & \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \geq 0 \end{aligned}$$

Solving this MC program requires the examination of all possible values of (λ_1, λ_2) . For convenience, we can assume a normalization for (λ_1, λ_2) ; that is,

$$\text{let } 0 \leq \lambda_1, \lambda_2 \leq 1 \text{ and } \lambda_1 + \lambda_2 = 1.$$

In such a way, we can locate all nondominated solutions when the values of (λ_1, λ_2) change between 0 and 1. Both systematical and interactive methods of solving MC linear programming will be reviewed in Chapter 3.

1.4 The MC² Linear Programming Problem

Both linear programming and MC linear programming models for the PC problem implicitly assume that there is only a single person, for example, the production manager in charge of decision making. This may not be true if we consider that a group of decision makers, including the president, chief financial officer, controller, and marketing manager, share the decision making responsibility. Extending a single decision maker to a multiple decision maker setting introduces the multiple resource availability level framework to modeling group decision making problems. The preference values of each involved manager on resource availability can be treated as an individual level of the availability. Then, the modified model has multiple discrete levels of resource availability. It is clear that the multiple decision makers may have different preferences, in terms of their backgrounds and positions in their firm, which result in inevitable conflicts. In addition to the tradeoff

between multiple criteria (recall Section 1.3), the “optimal solution” must reflect the compromise or tradeoffs among the multiple discrete levels of resource availability.

For simplicity, suppose that both production manager and financial manager are in charge of the PC production in the example of Section 1.2. We note that the resource availability level of the production manager is different from that of the financial manager. While the production manager views his/her availability level from the perspective of manpower and materials under control, the financial manager may decide the availability level according to the current cash flow and how much the company can borrow from a bank. Suppose the financial manager believes that the Trend Computing Company can produce 65,000 units of hardware for both type 1 and type 2 PC's, and can purchase 45,000 units of software. Then, the new extended data is given in Table 1.3.

Table 1.3 The PC Problem for MC²

Resource	Type 1	Type 2	Availability Level of Production Manager	Availability Level of Financial Manager
Hardware	3	2	40,000	65,000
Software	8	3	90,000	45,000
Profit (\$)	400	200		
Production Quantities	1	1		

In addition to considering the criteria space containing the tradeoffs of multiple criteria, the structure of MC² linear programming has a **constraint-level space** that shows all possible tradeoffs of resource availability levels (i.e., the preferences of the production manager and financial manager in the PC problem). To demonstrate the constraint-level space of the PC problem, let γ_1 be the weight parameter for the production manager and γ_2 be the weight parameter for the financial manager. Then, the new parametric constraint levels become:

$$40,000 \gamma_1 + 65,000 \gamma_2 \quad (\text{Hardware})$$

$$90,000 \gamma_1 + 45,000 \gamma_2 \quad (\text{Software})$$

An MC² (or two criteria and two constraint-level) linear program (M1.3) for the PC problem can be formulated as

$$\begin{array}{ll}
 \text{Max} & \lambda_1 (400 x_1 + 200 x_2) + \lambda_2 (x_1 + x_2) \\
 \text{Subject to} & 3 x_1 + 2x_2 \leq 40,000 \gamma_1 + 65,000 \gamma_2 \\
 & 8 x_1 + 3x_2 \leq 90,000 \gamma_1 + 45,000 \gamma_2 \\
 & x_1, x_2 \geq 0
 \end{array}$$

The corresponding matrix form of this MC² problem is

$$\begin{array}{l}
 \text{Max} \left(\lambda_1 \quad \lambda_2 \right) \begin{pmatrix} 400 & 200 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\
 \text{Subject to} \begin{pmatrix} 3 & 2 \\ 8 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 40,000 & 65,000 \\ 90,000 & 45,000 \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} \\
 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \geq 0
 \end{array}$$

Let $0 < \gamma_1, \gamma_2 \leq 1$ and $\gamma_1 + \gamma_2 = 1$ to normalize the constraint-level parameters. By using the *MC²-Simplex Method* that will be studied in Chapter 4, we can systematically locate a set of **potential solutions** to over all possible values of (γ_1, γ_2) and (λ_1, λ_2) . For any given value of (γ_1, γ_2) and (λ_1, λ_2) , there is a potential solution to optimize the PC problem. The structure of potential solutions provides a comprehensive scenario of all possible tradeoffs or compromises between the profit, the production quantities, the production manager, and the financial manager. The general managerial implications of applications will be discussed in Chapters 16, 17, and 21.

1.5 Further Remarks

Based on Dantzig's simplex method, linear programming has been rapidly developed in two directions since 1950's. One is the development of computer software that can solve large-scale linear programming problems. The early focus of the software development was

first on card-programmed calculators and then mainframes. Since 1980, the main focus has gradually turned to PC-based solvers. There are more than 30 different commercial software packages claiming the ability to solve large-scale problems. These software include stand-alone solvers and solvers integrated with modeling languages. Many of them are available in Windows environment and some of them have spreadsheet integration. The second direction is the real-world application of linear programming techniques. A recent survey showed that 85% of Fortune 500 companies have used linear programming in business decision making. Applications of linear programming cover transportation, agriculture, economic analysis, finance, energy industries, military, production and public policy.

Because MC linear programming involves the tradeoffs of multiple conflicting criteria, the development of solution algorithms has mainly been in three areas. The first applies the simplex method of linear programming to the MC structure. This results in the MC-simplex method that can *systematically* find all of nondominated solutions. The second uses an interactive approach to achieve one preferred solution that satisfies the decision maker's preference. Tremendous progress has been made on this approach over the last 20 years. There is much software being developed for academic usage and a few for the real-world applications. The third applies goal setting concepts to transform an MC problem into a linear goal programming problem. The latter can then be solved by the linear programming algorithms for minimizing the distance between the ideal solution and the desired solution (see Chapter 13). Like linear programming, the concept of MC linear programming has been attractive to practitioners. Since the 1970's, MC linear programming has been frequently applied in many real-world problems. The application of MC linear programming is parallel to that of linear programming. The reason is that whenever the practitioner uses linear programming and considers multiple criteria, the MC structure provides a good response.

In comparing linear programming and MC linear programming, both software development and application of MC² linear programming are currently in an "infancy" stage. The purpose of this book is to emphasize the potential and promise of MC² linear programming to academia and practitioners so that they can use this tool to solve the real-world decision problems.

Exercises

1.1 Plateau Com. developed three kinds of memory chips for next generation computers. Chip 1 and Chip 2 are made for PC, while Chip 3 is used for Workstation. The unit prices of Chip 1, Chip 2 and Chip 3 are \$150, \$100, and \$300, respectively. There are two different resources: labor and raw material involving in the production process. To produce a Chip 1, 5 hours of labor and 10 units of raw materials are needed. Each Chip 2 needs 4 hours of labor and 15 units of raw materials. A Chip 3 consumes 10 hours of labor and 8 units of raw materials. Suppose there are 240 labor hours available and 500 units of raw materials available from the production manager point of view.

Formulate a linear program to maximize the total revenue for the production manager.

1.2 In the LP problem 1.1, if the company wants to simultaneously maximize the total revenue and the total production quantities, formulate the MC linear program.

1.3 In addition to the MC problem 1.2, suppose the finance manager believes that 300 labor hours and 450 units of raw materials are available. Incorporate this information into the modeling process and formulate an MC² linear program.

1.4 InterWireless Co. is a major cellular phone producer. It has four overseas factories located in Beijing (China), Moscow (Russia), Seoul (Korea), and Tijuana (Mexico). These factories ship cellular phones back to three warehouses in the USA. Warehouses are located at Los Angeles, Omaha, and New York. The unit transportation costs from the factories to warehouses are given in Table 1.4:

Table 1.4

	Los Angeles	Omaha	New York
Beijing	\$10	\$15	\$20
Moscow	\$25	\$20	\$10
Seoul	\$8	\$12	\$18
Tijuana	\$2	\$9	\$12

Suppose the company has to consider the transportation damage rate (percentage) which is estimated as in Table 1.5:

Table 1.5

	Los Angeles	Omaha	New York
Beijing	.1%	.2%	.3%
Moscow	.5%	.3%	.15%
Seoul	.1%	.13%	.2%
Tijuana	.05%	.1%	.2%

The production process is influenced by the global economy. When the economy is in recession, the production amounts of cellular phone are 200 (Beijing), 150 (Moscow), 300 (Seoul), and 250 (Tijuana). The capacities of warehouses are 300 (Los Angeles), 200 (Omaha) and 400 (New York). When the economy is blooming, the production amounts change to 250 (Beijing), 200 (Moscow), 350 (Seoul), and 300 (Tijuana). The demands of warehouses are 350 (Los Angeles), 400 (Omaha) and 350 (New York).

If the prediction of the global economy is in recession, formulate linear programs to minimize the total transportation cost and minimize the total damage rate, respectively.

1.5 For the data of the problem 1.4, if the prediction of the global economy is blooming, formulate linear programs to minimize the total transportation cost and minimize the total damage rate, respectively.

1.6 Given the LP problem 1.4, we assume that the global economy is in recession. If the company wants to simultaneously minimize the total transportation cost and minimize the total damage rate, formulate the corresponding MC linear program.

1.7 Given the LP problem 1.5, if the company wants to simultaneously minimize the total transportation cost and minimize the total damage rate, formulate the MC linear program for the blooming global economy.

1.8 Based on problems 1.6 and 1.7, formulate an MC² linear program to study tradeoffs between the total transportation costs, the total damage rate, the bloom economy and the economy recession.

1.9 Midwest Electronics produces a popular VCR. The regular-time production cost of VCR is \$80 and the over-time production cost is \$100. If the products cannot be shipped out, the unit inventory holding cost per 6-month is \$2. However, when demand is higher than supply, the company may potentially lose customers. This is viewed as the shortage

cost, which is estimated as \$30 per customer. If the future economy is predicted as either “bloom” or “recession,” it will affect the production capacity of the company as well as the market demand. Assume the company’s aggregate production plan can be made for 6-month base. Then, the data to be used for minimizing the total production cost (regular plus over time) for the next year over fluctuations of the economy is given in Table 1.6. The data of Table 1.7 is used to minimize the total inventory holding cost for the next year. The data for minimizing the total shortage cost is provided in Table 1.8.

Table 1.6

Periods	First 6-month	Second 6-month	Capacity for Bloom Eco.	Capacity for Eco. Recess.
Initial Invent.	20	0	100	90
Regular 1	80	80	120	130
Over-time 1	100	100	50	30
Regular 2	80	80	200	210
Over-time 2	100	100	60	80
Demand for Bloom Eco.	230	300	550	
Demand for Eco. Recess.	340	200		540

Table 1.7

Periods	First 6-month	Second 6-month	Capacity for Bloom Eco.	Capacity for Eco. Recess.
Initial Invent.	0	2	100	90
Regular 1	0	2	120	130
Over-time 1	0	2	50	30
Regular 2	0	0	200	210
Over-time 2	100	100	60	80
Demand for Bloom Eco.	230	300	550	
Demand for Eco. Recess.	340	200		540

Table 1.8

Periods	First 6-month	Second 6-month	Capacity for Bloom Eco.	Capacity for Eco. Recess.
Initial Invent.	0	0	100	90
Regular 1	0	0	120	130
Over-time 1	0	0	50	30
Regular 2	30	0	200	210
Over-time 2	30	0	60	80
Demand for Bloom Eco.	230	300	550	
Demand for Eco. Recess.	340	200		540

If the prediction of the economy is in recession, write three linear programs to minimize the total production cost, minimize the total inventory holding cost, and minimize the total shortage cost, respectively.

1.10 Given the data of the problem 1.9, if the prediction of the global economy is blooming, write three linear programs to minimize the total production cost, minimize the total inventory holding cost, and minimize the total shortage cost, respectively.

1.11 Given the data of the problem 1.9, suppose that the economy will be in recession for the next year, write an MC linear program to simultaneously minimize the total production cost, the total inventory holding cost, and the total shortage cost.

1.12 Given the data of the problem 1.9, if the economy is predicted to be blooming in the next year, write an MC linear program to simultaneously minimize the total production cost, the total inventory holding cost, and the total shortage cost.

1.13 Given the data of the problem 1.9, formulate an MC² linear program that can describe the tradeoffs of the total production cost, the total inventory holding cost, and the total shortage cost over fluctuations of the bloom economy and the economy recession.

1.14 United Chemical Corporation has two divisions that process raw materials into intermediate or final products. Division 1 which is located in Kansas City, Missouri, manufactures two kinds of chemicals, called Products 1 and 2. Product 1 is an intermediate product and cannot be sold externally. It can, however, be processed further by Division 2 into a final product. Division 2, located in Atlanta, Georgia, manufactures Product 2 and finalizes Product 1 in Division 1. The executives (president, vice-president for production and vice-president for marketing) agree on three objectives: (1) maximize the overall company's profit, (2) maximize the market share goal of Product 2 in Division 1 and of Products 1 and 2 in Division 2, and (3) maximize the utilized production capacity of the company. The related data is given in Table 1.9. The vice-president for production and vice-president for marketing determine preferences of constraint levels. The president will make the final decision on the basis of the resulting compromises. The constraint levels are shown in Table 1.10.

From the production vice-president's point of view, formulate an MC linear program to simultaneously maximize the overall company's profit, the market share goal, and the utilized production capacity of the company.

1.15 Given the data of the problem 1.14, from the marketing vice-president's point of view, formulate another MC linear program to simultaneously maximize the overall company's profit, the market share goal, and the utilized production capacity.

1.16 Use the data of the problem 1.14 to formulate an MC² linear program that can explore compromises of the production vice-president and the marketing vice-president.

Table 1.9

	Division 1		Division 2	
Products	Product 1	Product 2	Product 1	Product 2
Unit Profit (\$)	-4	8	13	5
Market Price (\$)	0	40	46.2	38
Unit Utilization	4	4	3	2

Table 1.10

Constraints	Production V.P.	Marketing V.P.
Transfer product	0	100
Budget (\$)	45,000	40,000
Production capacity of Product 2 in Division 1	38,000	12,000
Production capacity of Product 1 in Division 2	45,000	50,000
Production capacity of Product 2 in Division 2	36,000	10,000

1.17 Global Investment Inc. plans to invest in the construction joint-venture projects. There are 3 possible projects under consideration. Each project will last 3 years. The decision makers consisting of the president, controller, and engineering manager have identified 3 evaluation criteria for the capital budgeting. The first is maximizing net profit value of each year. The second one is maximizing the growth (net sales increase) of the firm due to each project. The third is maximizing the flexibility of financing (inverse of debt to equity ratio). The data regarding these criteria are given in Tables 1.11, 1.12, and 1.13, respectively. The budget availability level data from the involved decision makers is listed in Table 1.14.

Formulate individual linear programs with integer variables for each evaluation criterion for every decision maker (Hint: there are 9 integer linear programs).

1.18 Use the data of the problem 1.17 to formulate three MC linear programs with integer variables for the president, controller, and engineering manager, respectively.

1.19 Create an MC² linear program with appropriate integer variables for solving this capital budgeting problem from the data of problem 1.17 (Details of integer MC² formulation can be found in Chapter 5).

Table 1.11

Cash Outlays for Each Year					
Projects	Net Value	Present	Year 1	Year 2	Year 3
1	\$20		\$24	\$16	\$6
2	16		12	10	2
3	11		8	6	6

Table 1.12

Net Sales Increase for Each Year			
Projects	Year 1	Year 2	Year 3
1	\$120	\$130	\$145
2	100	120	140
3	80	90	95

Table 1.13

Inverse of Debt to Equity Ratio for Each Year			
Projects	Year 1	Year 2	Year 3
1	.85	.90	.98
2	.90	.98	.95
3	.96	.97	.98

Table 1.14

Estimate Budget Availability			
Decision Makers	Year 1	Year 2	Year 3
President	\$50	\$30	\$30
Controller	40	45	30
Engineering Manager	55	40	20