

# Contents

Foreword	vii
Preface	ix
<b>Chapter 1 The Menu, The Map, and the Magic</b>	<b>1</b>
1.1 The Menu for the Grand Feast . . . . .	3
1.2 The Book's Audience . . . . .	5
1.3 The Map . . . . .	7
1.4 Reasoning in Review . . . . .	8
1.5 Reasoning by Computer versus Reasoning by a Person . . . . .	10
1.6 Obstacles to the Effective Automation of Reasoning . . . . .	13
1.6.1 Language . . . . .	14
1.6.2 Inference Rules . . . . .	15
1.6.3 Assignment Completion . . . . .	16
1.6.4 Strategy . . . . .	17
1.6.5 Redundancy . . . . .	17
1.6.6 Specific versus General Information . . . . .	18
1.6.7 Conclusion Retention . . . . .	18
1.6.8 Conclusion Generation . . . . .	19
1.6.9 Inadequate Focus . . . . .	20
1.6.10 Conclusion Repetition . . . . .	21
1.6.11 Redundancy-Control Transformations . . . . .	22
1.6.12 Size of Deduction Step . . . . .	22
1.6.13 Metarules for Program Use . . . . .	23
1.6.14 Indexing . . . . .	23
1.7 Paradigms for Reasoning and for Research . . . . .	23
1.8 The Future of Automated Reasoning . . . . .	26
<b>Chapter 2 Learning Logic by Example</b>	<b>29</b>
2.1 and, or, not, if-then (implies) . . . . .	29

2.2	A Language for Automated Reasoning Programs . . . . .	35
2.2.1	Predicates and Constants . . . . .	36
2.2.2	Variables . . . . .	37
2.2.3	Functions . . . . .	39
2.3	Combinations of or with and, Complex if-then, and DeMorgan's Laws . . . . .	44
2.4	Assumptions and Axioms, Types of Reasoning, and Proof . . . . .	45
2.4.1	Assumptions and Axioms . . . . .	46
2.4.2	Types of Reasoning, Inference Rules . . . . .	47
2.4.3	Proof . . . . .	50
2.5	Summary . . . . .	56
<b>Chapter 3 Automated Reasoning in Full</b>		<b>63</b>
3.1	Logic . . . . .	64
3.1.1	and . . . . .	64
3.1.2	or . . . . .	64
3.1.3	not . . . . .	65
3.1.4	if-then and implies . . . . .	65
3.1.5	is-equivalent-to . . . . .	65
3.1.6	Relationships and Laws in Logic . . . . .	65
3.2	A Language Understood by an Automated Reasoning Program . . . . .	67
3.2.1	Variables . . . . .	68
3.3	Submitting a Problem to a Reasoning Program . . . . .	70
3.3.1	Assumptions and Axioms . . . . .	70
3.3.2	Special Facts and the Special Hypothesis . . . . .	70
3.3.3	Denial of the Goal or Theorem . . . . .	71
3.4	Inference Rules . . . . .	72
3.4.1	Unification . . . . .	72
3.4.2	Binary Resolution . . . . .	73
3.4.3	UR-Resolution . . . . .	74
3.4.4	Hyperresolution . . . . .	75
3.4.5	Paramodulation . . . . .	76
3.4.6	Other Inference Rules . . . . .	78
3.5	The Empty Clause . . . . .	79
3.6	Proof by Contradiction . . . . .	79
3.7	Demodulation . . . . .	80
3.8	Subsumption . . . . .	82
3.9	Strategy . . . . .	84
3.9.1	The Set of Support Strategy . . . . .	84
3.9.2	Weighting . . . . .	85
3.9.3	Unit Preference Strategy . . . . .	85
3.9.4	Other Strategies . . . . .	86

3.10	An Automated Reasoning Program in Action . . . . .	87
3.11	OTTER and Earlier Automated Theorem-Proving Programs . . . . .	89
3.11.1	The Basic Argonne Paradigm . . . . .	93
3.11.2	A Sequential Theorem-Proving Algorithm . . . . .	94
3.11.3	Implementing the Algorithm on Shared-Memory Multiprocessors . . . . .	95
3.11.4	Implementing the Algorithm on Distributed-Memory Machines . . . . .	98
3.12	Answers to Exercises . . . . .	100
<b>Chapter 4 Logic Circuit Design</b>		<b>107</b>
4.1	Introduction to Logic Circuit Design . . . . .	107
4.1.1	Components of Logic Circuits . . . . .	108
4.1.1.1	AND, OR, and NOT gates . . . . .	108
4.1.1.2	Other Basic Components . . . . .	109
4.1.2	Specifications . . . . .	110
4.1.3	Problems for the Circuit Designer . . . . .	111
4.2	Circuit Design Using Demodulation . . . . .	112
4.2.1	Designing from Functional Specifications . . . . .	112
4.2.2	Designing from Descriptive Sentences . . . . .	116
4.2.3	Designing from Tables . . . . .	117
4.2.4	Advantages and Disadvantages of the Demodulation Approach . . . . .	119
4.3	Circuit Design Using Hyperresolution . . . . .	119
4.3.1	The OUTPUT Predicate . . . . .	120
4.3.2	Construction Rules . . . . .	120
4.3.3	Examples . . . . .	121
4.3.4	Achievable Signals and the OUTPUT Predicate . . . . .	124
4.4	Solution to the Two-Inverter Puzzle . . . . .	124
4.4.1	Some New Complexities . . . . .	125
4.4.2	The First Formulation . . . . .	126
4.4.3	Reversions . . . . .	131
4.4.4	Solving the Puzzle Faster . . . . .	134
4.5	Multivalued Logic Design Using Negative Hyperresolution . . . . .	137
4.5.1	T-gates . . . . .	138
4.5.2	Representation of Multivalued Logic Functions . . . . .	138
4.5.3	Construction Rules . . . . .	139
4.5.4	Negative Hyperresolution . . . . .	139
4.5.5	An Example . . . . .	140
4.6	Answers to Exercises . . . . .	142
<b>Chapter 5 Logic Circuit Validation</b>		<b>147</b>
5.1	What Is Validation? . . . . .	147
5.2	Simple Example . . . . .	148

- 5.3 Important Features of the Example . . . . . 150
  - 5.3.1 Validation as Language Translation . . . . . 150
  - 5.3.2 Canonicalization . . . . . 151
  - 5.3.3 Simplification . . . . . 152
  - 5.3.4 Commutativity-Type Demodulators and Lexical Ordering . . . . . 152
- 5.4 A More Complex Example . . . . . 153
- 5.5 Validating an Adder . . . . . 157
  - 5.5.1 The One-Bit Full Adder . . . . . 157
  - 5.5.2 Combining Validated Subcircuits . . . . . 161
- 5.6 Answers to Exercises . . . . . 163
  
- Chapter 6 Research in Mathematics . . . . . 165**
- 6.1 A Simple Example . . . . . 166
- 6.2 A Simple Example Revisited . . . . . 171
  - 6.2.1 The Set of Support Strategy and the Simple Example . . . . . 174
  - 6.2.2 Weighting Applied to the Example . . . . . 178
  - 6.2.3 Demodulation, Canonicalization, and Simplification . . . . . 180
  - 6.2.4 Subsumption and Redundancy . . . . . 182
- 6.3 Answering Open Questions . . . . . 187
- 6.4 Model and Counterexample Generation . . . . . 190
- 6.5 Representable Concepts: Easy and Difficult . . . . . 197
- 6.6 Review . . . . . 199
- 6.7 Using OTTER to Find Shorter Proofs . . . . . 200
- 6.8 Answers to Exercises . . . . . 205
  
- Chapter 7 Research in Formal Logic . . . . . 211**
- 7.1 Equivalential Calculus . . . . . 212
- 7.2 A Simple Example . . . . . 213
- 7.3 Imposing Knowledge and Intuition . . . . . 215
- 7.4 Answering Open Questions in Equivalential Calculus . . . . . 217
  - 7.4.1 Finding Useful Notation . . . . . 219
  - 7.4.2 Suggesting Conjectures . . . . . 220
- 7.5 Propositional Calculus . . . . . 221
  - 7.5.1 Proof by Analogy and Other Techniques for Solving Hard Problems 222
  - 7.5.2 Seeking Shorter Proofs . . . . . 224
  - 7.5.3 Finding New Axiom Systems . . . . . 228
- 7.6 Combinatory Logic . . . . . 230
  - 7.6.1 Constructing Objects . . . . . 231
  - 7.6.2 Successes in Combinatory Logic . . . . . 233
- 7.7 Using OTTER to Seek Shorter Proofs in Two-Valued Sentential Calculus 234
- 7.8 Using OTTER to Construct Combinations . . . . . 242
- 7.9 Answers to Exercises . . . . . 248

<b>Chapter 8</b>	<b>The Formal Treatment of Automated Reasoning</b>	<b>255</b>
8.1	First-Order Predicate Calculus . . . . .	255
8.1.1	The Language of FOPC . . . . .	256
8.1.2	The Semantics of FOPC . . . . .	260
8.1.3	The Calculus of FOPC and the Concept of Proof . . . . .	262
8.2	The Clause Language . . . . .	265
8.3	Inference Rules . . . . .	270
8.3.1	Binary Resolution . . . . .	274
8.3.2	Refutation Completeness . . . . .	278
8.4	Strategy . . . . .	285
8.5	Other Inference Rules . . . . .	291
8.6	Subsumption . . . . .	298
8.7	Demodulation . . . . .	299
8.8	Answers to Exercises . . . . .	301
<b>Chapter 9</b>	<b>Wos's Biased Guide for the Effective Use of OTTER</b>	<b>309</b>
9.1	Overview to Option Selection . . . . .	312
9.2	Option Choosing . . . . .	313
9.2.1	Representation . . . . .	313
9.2.2	Strategy . . . . .	315
9.2.2.1	Set of Support Strategy . . . . .	315
9.2.2.2	Weighting . . . . .	317
9.2.2.3	Level Saturation . . . . .	320
9.2.2.4	Ratio Strategy . . . . .	321
9.2.2.5	Tail Strategy and Recursive Tail Strategy . . . . .	322
9.2.2.6	Resonance Strategy and Resonance-Restriction Strategy . . . . .	324
9.2.2.7	Subtautology . . . . .	326
9.2.3	Hot List Strategy and Dynamic Hot List Strategy . . . . .	326
9.2.4	Inference Rules . . . . .	327
9.2.4.1	UR-Resolution . . . . .	327
9.2.4.2	Hyperresolution . . . . .	328
9.2.4.3	Paramodulation . . . . .	329
9.2.4.4	Hyperparamodulation . . . . .	330
9.2.4.5	Binary Resolution . . . . .	330
9.2.5	Subsumption, for Purging Redundant Information . . . . .	330
9.2.6	Demodulation, for Simplification and Canonicalization . . . . .	332
9.2.7	List Usage . . . . .	334
9.3	An Emphasis on Experimentation . . . . .	338
9.4	Four Useful Input Files . . . . .	339
9.4.1	A Puzzle to Solve . . . . .	339
9.4.2	A Theorem to Prove . . . . .	342
9.4.3	A Shorter Proof to Find . . . . .	343

<b>Chapter 10 An Author's Appraisal of His Papers</b>	<b>347</b>
10.1 On Commutative Prime Power Subgroups of the Norm . . . . .	349
10.2 The Unit Preference Strategy in Theorem Proving . . . . .	351
10.3 Efficiency and Completeness of the Set of Support Strategy in Theorem Proving . . . . .	355
10.4 Automatic Generation of Proofs in Mathematics . . . . .	364
10.5 The Concept of Demodulation in Theorem Proving . . . . .	366
10.6 Paramodulation and Set of Support . . . . .	371
10.7 Axiom Systems in Automatic Theorem Proving . . . . .	375
10.8 Paramodulation and Theorem-Proving in First-Order Theories with Equality . . . . .	377
10.9 Maximal Models and Refutation Completeness . . . . .	386
10.10 A Theorem-Proving Language for Experimentation . . . . .	388
10.11 Unit Refutations and Horn Sets . . . . .	388
10.12 Problems and Experiments for and with Automated Theorem-Proving Programs . . . . .	389
10.13 Complexity and Related Enhancements for Automated Theorem-Proving Programs . . . . .	391
10.14 Unnatural Attack on the Structure Problem for the Free Jordan Ring on 3 Letters . . . . .	395
10.15 Automated Generation of Models and Counterexamples . . . . .	396
10.16 Hyperparamodulation: A Refinement of Paramodulation . . . . .	399
10.17 Semigroups, Antiautomorphisms, and Involutions . . . . .	402
10.18 An Automated Reasoning System . . . . .	404
10.19 Solving Open Questions with an Automated Theorem-Proving Program	405
10.20 Demodulation and Related Tricks . . . . .	406
10.21 Automated Theorem-Proving 1965–1970 . . . . .	408
10.22 Shortest Single Axioms in Equivential Calculus . . . . .	408
10.23 Automated Reasoning: Real Uses and Potential Uses . . . . .	411
10.24 Equivential Calculus and Infinite Domains . . . . .	412
10.25 Open Questions Solved with the Assistance of AURA . . . . .	413
10.26 <i>The Linked Inference Principle, II: The User's Viewpoint</i> . . . . .	414
10.27 Achievements in Automated Reasoning . . . . .	417
10.28 Automated Reasoning Programs: How They Work . . . . .	417
10.29 Automated Reasoning . . . . .	418
10.30 What Is Automated Reasoning? . . . . .	419
10.31 Automating Reasoning . . . . .	419
10.32 Job-Shop Scheduling Using Automated Reasoning . . . . .	420
10.33 Negative Paramodulation . . . . .	421
10.34 Set Theory in First-Order Logic: Clauses for Gödel's Axioms . . . . .	423
10.35 Resolution, Binary . . . . .	425
10.36 Finding Sages in Combinatory Logic . . . . .	426

10.37 Challenge Problems Focusing on Equality and Combinatory Logic . . .	428
10.38 Automated Theorem Proving and Logic Programming: A Natural Symbiosis . . . . .	431
10.39 Subsumption, A Sometimes Undervalued Procedure . . . . .	432
10.40 Applications of Automated Reasoning . . . . .	433
10.41 Meeting the Challenge of Fifty Years of Logic . . . . .	434
10.42 The Absence and the Presence of Fixed Point Combinators . . . . .	437
10.43 Automated Reasoning Contributes to Mathematics and Logic . . . . .	438
10.44 Automated Reasoning and Bledsoe’s Dream for the Field . . . . .	439
10.45 A Logical Basis for the Automation of Reasoning: Case Studies . . . . .	440
10.46 The Linked Inference Principle, I: The Formal Treatment . . . . .	441
10.47 The Application of Automated Reasoning to Questions in Mathematics and Logic . . . . .	443
10.48 Automated Reasoning and Enumerative Search . . . . .	443
10.49 Benchmark Problems in Which Equality Plays the Major Role . . . . .	444
10.50 Automated Deduction with Condensed Detachment . . . . .	445
10.51 Single Axioms for Exponent Groups . . . . .	447
10.52 The Impossibility of the Automation of Logical Reasoning . . . . .	449
10.53 The Kernel Strategy and Its Use for the Study of Combinatory Logic .	450
10.54 Automated Reasoning Answers Open Questions . . . . .	453
10.55 The Field of Automated Reasoning . . . . .	456
10.56 The Resonance Strategy . . . . .	456
10.57 Searching for Circles of Pure Proofs . . . . .	461
10.58 The Power of Combining Resonance with Heat . . . . .	463
10.59 The Hot List Strategy . . . . .	466
10.60 OTTER and the Moufang Identity Problem . . . . .	471
10.61 Automating the Search for Elegant Proofs . . . . .	472
10.62 Otter: The CADE-13 Competition Incarnations . . . . .	478
10.63 Programs That Offer Fast, Flawless, Logical Reasoning . . . . .	480
10.64 Hints and Answers . . . . .	480
10.64.1 Hints . . . . .	481
10.64.2 Answers . . . . .	482

**Chapter 11 Open Questions, Hard Problems, Intriguing Challenges** 493

11.1 Distinctions, Limits, and Overview . . . . .	495
11.2 Questions and Problems from Combinatory Logic . . . . .	497
11.3 Questions and Problems from Mathematics . . . . .	505
11.3.1 Group Theory . . . . .	506
11.3.2 Ring Theory . . . . .	507
11.3.3 Robbins Algebra . . . . .	508

11.3.4	More Algebra . . . . .	508
11.4	Questions and Problems from Logic . . . . .	510
11.4.1	Equivalential Calculus . . . . .	510
11.4.2	Propositional Logic . . . . .	514
11.4.3	Many-Valued Sentential Calculus . . . . .	517
<b>Chapter 12</b>	<b>Epilogue and After-Dinner Liqueur</b>	<b>521</b>
<b>Appendix A</b>	<b>Featuring Input Files, Proofs, and Output File Fragments</b>	<b>523</b>
References		561
Index		571