

Contents

1. Introduction	1
1.1. Invariant Structures Everywhere	1
1.1.1. Resonance Structures in Celestial Mechanics	2
1.1.2. Cellular, Spiral, Vortex and Crystal Structures	4
1.1.3. Fractals	9
1.2. Dynamical Systems	11
1.2.1. Attractors	13
1.2.2. Invariant Tori	15
1.3. Discrete Dynamical Systems — Maps	16
I Computer-Generated Invariant Sets	19
2. Description of WinSet Program	21
2.1. Installation	21
2.2. Basics of WinSet	21
2.2.1. First Run of WinSet	22
2.2.2. Using the Mouse and the Keyboard	23
2.3. Your First Invariant Set	24
2.4. WinSet Menu	25
2.5. Three-Dimensional Objects	35
2.6. Diffusion Equations	36
2.7. Defining Your Own Equations	41
3. List of the Built-in Equations, Maps and Fractals of WinSet. Main Invariant Sets of WinSet	45
3.1. Maps	45
3.1.1. Cathala Map	45
3.1.2. Chirikov Map	45
3.1.3. Henon Maps	47

3.1.4.	Julia Map	47
3.1.5.	Mira and Gumowski Maps	48
3.1.6.	Zaslavsky Map	52
3.2.	Fractals	52
3.2.1.	Coloring the Fractals	52
3.2.2.	Julia Fractals	53
3.2.3.	Mandelbrot Fractal	56
3.2.4.	Mira Fractals	56
3.2.5.	Newton Fractal	57
3.3.	Ordinary Differential Equations (ODE)	57
3.3.1.	Brusselator	57
3.3.2.	Chua Equations	58
3.3.3.	Duffing Type Equations	58
3.3.4.	Hamiltonian Systems on Torus	62
3.3.5.	Henon-Heiles Model	62
3.3.6.	Henon-Heiles Type Equations	63
3.3.7.	Kepler Equation	63
3.3.8.	Kolmogorov-Volterra Equations	63
3.3.9.	Lorenz Equations	65
3.3.10.	Motion of Particle in Gravitation Field	65
3.3.11.	Pendulum Equations	67
3.3.12.	Equations with Quadratic Nonlinearity	69
3.3.13.	Roessler Equations	70
3.3.14.	Volterra Equations	70
3.4.	Diffusion Equations (PDE)	71
3.4.1.	Brusselator Model	71
3.4.2.	Fitz Hugh-Nagumo Equations	72
3.4.3.	Lengyel-Epstein Model (CIMA)	72
3.4.4.	Semi-Discrete Equation	73
3.5.	Numerical Methods Used by WinSet	74

II Mathematical Description of Invariant Sets 77

4.	Invariant Sets in Hamiltonian Mechanics 79
4.1.	Generalities 79
4.2.	Invariant Sets of Hamiltonian Systems with One Degree of Freedom . 82
4.3.	Invariant Sets of Hamiltonian Systems with $3/2$ Degrees of Freedom . 90
4.3.1.	Poincaré Map 90
4.3.2.	Analytic Study 93

4.3.3.	Duffing Type Equations	98
4.3.4.	Pendulum Type Equation	101
4.3.5.	Systems on the Torus	104
4.3.6.	Kepler Equation	104
4.4.	Invariant Sets of Hamiltonian Systems with Two Degrees of Freedom.	106
4.4.1.	Henon-Heiles Type Systems	106
4.4.2.	Invariant Sets in the Dynamics of a Solid	107
5.	Area-Preserving Maps	111
5.1.	Chirikov Map	111
5.2.	Gumowski & Mira Map	113
5.3.	Henon Map	113
5.4.	Zaslavsky Map	118
6.	Non-Conservative Systems	123
6.1.	Characteristics of Chaotic Dynamics	124
6.1.1.	Characteristics which do not Use Measure	125
6.1.2.	Measure-Theoretic Characteristics of the Attractor	127
6.1.3.	Power Spectrum of an Observable	129
6.2.	Self-Oscillations	130
6.2.1.	Some Technical Transformations	132
6.2.2.	Qualitative Behavior of Solutions in an Individual Cell	134
6.2.3.	Behavior of Solutions near Separatrices of the Unperturbed System	136
6.2.4.	Van der Pole – Duffing Type Equations	137
6.2.5.	Pendulum Type Equations	138
6.2.6.	Brusselator Equation	140
6.2.7.	Three-Dimensional Systems	143
6.3.	Resonances and Synchronization	145
6.3.1.	Theoretical Analysis of Quasi-Hamiltonian Systems with 3/2 Degrees of Freedom	146
6.3.2.	Characteristics of Chaotic Dynamics for Systems with 3/2 Degrees of Freedom	160
6.3.3.	Theoretical Analysis of Quasi-Hamiltonian Systems with Two Degrees of Freedom	162
6.3.4.	Examples	169
6.4.	Parametric Resonances	177
6.4.1.	General Results	178
6.4.2.	Example 1	186
6.4.3.	Example 2	190

6.5. Strange Attractors in Three-Dimensional Systems	198
6.5.1. Lorenz System	198
6.5.2. Roessler System	205
6.5.3. Chua System	205
7. Non-Conservative Maps	211
7.1. One-Dimensional Maps	213
7.2. Two-Dimensional Non-Conservative Maps	216
7.2.1. One-Dimensional Complex Rational Endomorphisms	216
7.2.2. Fractals	219
7.2.3. Non-Invertible Mira Maps and their Fractals	226
7.2.4. Henon Map	232
8. Diffusion Equations	235
8.1. Parabolic Equations	236
8.1.1. One-Component Models	236
8.1.2. Two-Component Model	238
8.2. Semi-Discrete Approximation	243
8.2.1. Approximation of Equation (8.1)	244
8.2.2. Approximation of the Basic Multi-Component Models	245
8.3. Semi-Discrete Diffusion Equations	245
Bibliography	251