space with Ω discrete states. Such times are effectively infinite (exceeding the Age of the Universe) once the number of particles is of order ten to a hundred. Despite the formal periodicity it is quite possible to describe Lyapunov instability, the sensitivity to small perturbations called "chaos", using the bit-reversible approach.

1.5 Instability and Chaos

Turbulence has long been singled out as a specially "difficult" subject. This characterization of turbulence has arisen from the continuing failure of attempts to predict, or at least to understand, the long-time behavior of complex flows, such as our weather, despite the well-recognized importance of the task. Turbulent instability occurs whenever the decay rate associated with fluid deformations—changes in shape—is sufficiently small. In 1963 Lorenz described his efforts to continue the numerical solution of his now-famous set of three differential equations[§], starting out from intermediate values. He found that this second solution failed to agree with his original one after a fairly short time. Further investigation showed that the mechanism for the disagreement was the *exponentially* unstable loss of information, with the precision required to reproduce a solution of fixed accuracy increasing *exponentially* with the required time, corresponding to the required number of decimal *digits* or binary *bits* increasing *linearly* with time. With Lorenz' work it became "widely known" (to experts) that *most* flows contain this same sensitivity, "Lyapunov instability", to small changes in initial conditions.

Quite typically, both microscopic and macroscopic equations of motion are Lyapunov unstable, meaning that their solutions are very sensitive to small perturbations, so sensitive that such perturbations grow *exponentially* fast, in time. Though it is completely deterministic, and in principle reproducible, the chaos which characterizes Lyapunov instability means that particular precise initial conditions are not a useful concept. This is just as well, inasmuch as the concept of a completely isolated system has no sound basis in physics, where everyday gravitational forces have infinite range. On the other hand, *gross* characteristics of initial conditions *do* describe the spatial variations of macroscopic features, such as the temperature or

---

[§] $\dot{x} = -\sigma(x-y)$; $\dot{y} = \mathcal{R}x - y - xz$; $\dot{z} = xy - bz$. See Section 4.9.1 for more details.

velocity field. The gross features lead to the reproducible averaged behavior described by the macroscopic physical theories. They also suggest using statistical ensembles to describe the time development of similar systems.

The details of microscopic initial conditions cannot be to blame for irreversible behavior, for the time-reversibility property of microscopic theories is completely independent of these conditions. The only likely possibility for breaking the apparent symmetry of future and past solutions lies in their relative *stabilities.* These stabilities can be analyzed in detail in terms of the Lyapunov spectrum, as is discussed in detail in Chapter 7. The Lyapunov exponents which make up the spectrum describe the *global* time-averaged rates of growth, or decay, of perturbations in the initial conditions. The complete spectrum gives a description of these rates for *all* perturbation directions, not just the most-rapidly-growing one. For Newton's equations of motion both the global *and* the local spectra proceeding forward in time are exactly equal to their time-reversed global or local analogs, regressing *backward*, in time.

Krylov emphasized, that for this reason—the symmetry linking the past and the future—irreversibility cannot be understood from the standpoint of Newton's equations. More recently, Smale and Ruelle have echoed this view, suggesting that the description of irreversible processes requires new generalizations of the classical equations of motion. With the advent of computers making simulations possible, these generalizations were not long in coming. Analyses of the results have revealed an interesting ubiquitous breaking of time symmetry. *Typically*, the *forward* nonequilibrium computer trajectory is *less* sensitive to perturbations—and thus *more* stable—than is the time-reversed backward one. This difference in sensitivity leads both to a symmetry breaking and to a simple geometric understanding of the irreversibility which lies all around us. It also gives rise to singular *fractal* distributions. These are distributions which have no well-defined gradients. Locally smooth in some directions while wildly singular in others, fractals display a pervasive power-law structure on *all* length scales. We will consider the first of many such examples in the next Chapter.

Loschmidt emphasized the paradoxical aspect of time-reversible Newtonian trajectories. And classical mechanics, as originated by Newton, but generalized to include boundaries, constraints, and driving forces, will be the main focus of our interest in reversibility. In relativity theory, electromagnetism, and quantum mechanics, time reversibility is less apparent in the fundamental equations, but is nevertheless present. Schrödinger was intrigued by this problem too. In a 1931 lecture described in his *Science, Theory, and Man* he publicized Exner's sceptical criticism of the view that conventional perfectly-deterministic, and time-reversible, classical mechanics is the only possible model describing "classical" phenomena. So long as energy and momentum are conserved in collisions, a small stochastic contribution to the dynamics *could* also be present, accounting for irreversibility. Exner's explanation, though technically possible, seems implausible, because it fails Occam's test of simplicity. Apart from integer algorithms, like Levesque and Verlet's, finite precision results in computational roundoff error. It seems to me very unlikely that stochastic low-level noise differs in any *significant* respect from this computational error.

1.6 Simple Explanations of Complex Phenomena

Time itself can be viewed as a puzzle, but I choose not to do so. And I also choose to ignore the couplings between mass, space, and time revealed by relativity. For me, Time is a primitive intuitive notion, like Space and Place. I think of time in purely-classical nonrelativistic terms. Time's passage can then be quantified through any periodic motion. It is the result of experience that the exact nature of that motion is immaterial. The difficulties involved in finding a precise and general definition of "time" are not important to an understanding of time reversibility. Simplicity dictates an understanding based on nonrelativistic classical concepts.

At the end of the nineteenth century, and again, toward the middle of the twentieth, some vocal physicists looked forward to finding a unified view of nature. In addition to linking our sensations to the physical world, through understanding consciousness, such a unified view would also require a consistent mathematical description of complex phenomena. The emergence of chaos and complexity renders such a goal obsolete. Gödel showed that most interesting purely-mathematical theories are intrinsically incomplete, unable to decide the truth or untruth of definite statements.

The premature announcements, around 1900 and again around 1950, that "classical mechanics is dead" evidently stemmed from this same obsolete viewpoint of a "complete" theory. If there were some complete and unified view of nature, then more-specialized and restricted special cases of it could perhaps be thought of as second-rate, even if their structure were simpler. Chaos limits the ability of the various theories to overlap.