Preface

This book addresses selected practical applications and recent developments in the areas of quantitative financial modeling in derivatives instruments. While the primary scope of this book is the fixed-income market (with further focus on the interest rate market), many of the methodologies presented can also be applied to other financial markets, such as the equity, credit, and foreign exchange markets.

This book is written from the point of view of financial engineers or practitioners, and, as such, it puts more emphasis on the practical applications of financial mathematics in the real market than the mathematics itself with precise (and often tedious) technical conditions. It does not aim to be comprehensive; rather, it focuses on selected areas based on the authors' experience and expertise. It also attempts to combine economic and financial insights with mathematics and modeling so as to help the reader develop intuitions.

This book also presents various exploratory (and possibly thought-provoking) ideas and practical issues that are beneficial to academics and practitioners as the leads for further research. In terms of a popular saying that "the engineers have problems, but no solutions, and the mathematicians have solutions, but no problems," we are trying to supply some solutions, as well as some problems as part of the book.

While the primary targeted readers are Quant modelers or those who are studying to become Quant modelers, many parts of this book are also beneficial to Quant developers, Quant structurers, Quant traders, and possibly senior managers. This book assumes that the reader is familiar with the basics of stochastic calculus and derivatives modeling.

The following are a few examples of the practical applications of financial mathematics in the real market covered by this book.

For instance, this book addresses various practical applications of modeling, such as martingale arbitrage modeling under real market situations (such as using the correct risk-free interest rate, revised put-call parity, defaultable derivatives, hedging in the presence of the volatility skew and smile, secondary model calibration for handling the unhedgeable variables, models for pricing and models for hedging). This book also presents practical numerical algorithms for the model implementation, such as martingale interpolation and resampling for enforcing discrete martingale relationships *in situ* in numerical procedures, modeling of the volatility skew and smile, and a nonexploding bushy tree (NBT) technique for efficiently solving non-Markovian models under the backward induction framework. In addition, this book also covers the basics of the interest rate market, including various yield curve modeling, such as the well-known Orthogonal Exponential Spline (OES) model.

This book also discusses various trading and structuring strategies, including those for market making and those for proprietary trading including hedging, yield/coupon enhancement, funding cheapening, and leveraging.[1] For instance, this book touches upon various popular derivatives instruments, such as TArget Redemption Notes (TARNs), (callable, knock-out, or TAR) inverse floaters, (callable, knock-out, or TAR) capped floaters, (callable, knock-out, or TAR) CMS (Constant Maturity Swaps) spread floaters, (callable, knock-out, or TAR) (CMS spread) range accrual notes, (callable, knock-out, or TAR) Snowballs and Snowbears, (callable, knock-out, or TAR) Power Reverse Dual Currency (PRDC) notes, some credit/IR/FX hybrids (such as CCDS (contingent CDS) and credit extinguishers), as well as asset swaps or repackaging. As we shall see, these strategies essentially are trading activities between the arbitrage-free probability measure and the real world probability measure or between the arbitrage models and the forecasting models.

Furthermore, this book addresses the counterparty credit risk modeling, pricing, trading/arbitraging strategies, and risk management that are recent developments and are of increasing importance. The counterparty credit risk comes from the fact that as one enters into an OTC (over-the-counter) derivatives transaction, one (implicitly or explicitly) grants one's counterparty an option to default and, at the same time, one also receives an option to default oneself.

[1] It is important to point out that not all the trading strategies are necessarily legal in all jurisdictions and, obviously, one should use them within the legal boundaries.

The term of counterparty credit risk may remind the reader of a back office risk management functionality or a treasury function. We emphasize that it should be an integral part of the front office derivatives pricing and hedging activities. This is the approach that we shall adopt in this book. With this approach, a large part of the counterparty credit risk becomes the market risk that can be traded and hedged with market oriented trading strategies (in addition to the classical strategies for counterparty credit risk management).

In essence, pricing the counterparty credit risk is the same as pricing defaultable derivatives or the default premium in OTC derivatives instruments, which is a relatively new area of derivatives modeling and trading. In some cases, the counterparty credit risk is more significant than any other risks and a derivatives instrument becomes a hybrid derivatives instrument with credit or a compound credit derivatives instrument. For instance, a plain vanilla interest rate (IR) swap becomes a defaultable IR swap. Also, in the prevailing credit derivatives, to price a credit default swap (CDS) is essentially to price the default premium in the underlying bond, whereas to price the counterparty credit risk is to price the default premium of the default protection seller of the CDS or, more precisely, the premium of the joint default of the underlying asset and the default protection seller. Additionally, while cash settled IR swaptions and physically settled IR swaptions can have identical prices from a non-defaultable IR model, their associated counterparty default premiums can be significantly different. Furthermore, the exercise boundaries of Bermudan and American options can be significantly changed by the counterparty default risk.

For IR derivatives, in particular, the counterparty credit risk modeling is essentially the same as the IR/Credit hybrid modeling.

The PV (present value) of the counterparty credit risk can be a significant percentage of the overall P&L (profit and loss) of the derivatives desks and can reach many hundreds of millions of dollars for a top investment bank. The PV of the counterparty credit risk needs to be actively hedged and risk managed, just like other risks in derivatives trading.

There are also arbitrage strategies that one can employ to monetize the counterparty credit risk by bringing in cashflows in some cases and by reducing the outgoing cashflows in some other cases. These strategies can also be applied to achieve trade price cheapening to one's counterparties and, thus, can bring in more business. In other words, while one usually needs to charge a premium from a low credit rated counterparty due to the counterparty's default risk, one can actually structure a trade whereby one pays a premium to one's counterparty due to the counterparty's likelihood of default. The reader may want to think about how to structure such a trade and we shall come back to this point later on.

All these have created and will likely create more trading opportunities in the market place, especially after the implementation of the Basel II Accord, which will be an important driving factor for the development of the credit hybrid market and modeling. Without proper modeling and management of the counterparty credit risk, one is missing a significant part of the big picture of the OTC derivatives modeling and business.

As we shall see that one challenging and interesting aspect of pricing the counterparty default risk premium for a trade is that it depends on the portfolio that the trade is in and has non-linear portfolio effects due to the netting agreement and the default treatment conventions.

This is the major motivation for us to divide derivatives models into separate categories, such as trade-level or *micro* models and portfolio level or *macro* models (where the portfolio information acts as an input). Most of the prevailing derivatives models are trade-level or *micro* models, focusing on the modeling of a specific trade regardless of the portfolio it is in. However, as we shall see, there are several categories of risks that (directly or indirectly) contribute to the price (or the funding cost) of a specific trade have non-linear portfolio effects. Aside from the counterparty credit risk, other examples of these types of risks include, the unexpected risks, such as VAR (Value at Risk) and PE (Potential exposure), and the risks associated with the un-hedgeable variables.

More specifically, this book is divided into two parts: Part I - Theory and Applications of Derivatives Modeling and Part II - Interest Rate Market Fundamentals and Proprietary Trading Strategies. Various practical numerical implementation techniques and selected topics in financial stochastic calculus are also presented.

Part I provides a framework for general derivatives modeling, fixed-income derivatives modeling (with emphasis on interest rate derivatives modeling), and counterparty credit risk modeling.

For general derivatives modeling, it covers Harrison-Pliska, martingale arbitrage modeling framework, martingale arbitrage modeling under real market conditions (such as using the correct risk-free interest rate, revised put-call parity, defaultable derivatives, hedging in the presence of the volatility skew and smile, secondary model calibration for handling the un-hedgeable variables, models for pricing and models for hedging), and martingale interpolation and resampling for enforcing discrete martingale relationships *in situ* in numerical procedures. Black-Scholes framework and its extensions derived from martingale modeling are discussed. The impact of volatility skews and smiles on hedging is also discussed.

For interest rate derivatives modeling, it covers various interest rate term structure models, such as the Heath-Jarrow-Morton (HJM) framework, the Brace-Gatarek-Musiela (BGM) market model, and Markovian HJM models, such as the multi-factor Ritchken-Sankarasubramanian (RS) model. The details of the nonexploding bushy tree (NBT) technique for efficiently solving multi-factor full yield curve interest rate models, as well as the tree and Monte Carlo based numerical techniques in general are discussed with various volatility structures such as lognormal, shifted lognormal, Constant Elasticity of Variance (CEV), as well as stochastic volatilities.

For counterparty credit risk modeling, it provides a review of counterparty credit risk and covers how to price the counterparty default options, how to incorporate such risk into the price (or spread) of the OTC derivatives, how to derive and apply various trading and structuring strategies, and how to manage the risk associated with such options. It also touches upon general credit risk modeling and pricing, as well as a risky market model for credit spread modeling.

Part II covers selected areas of the fixed income market, such as the introduction to some basic interest rate products, the yield curve construction and modeling, including the well-known Orthogonal Exponential Spline (OES) model. It also covers proprietary trading strategies using statistical arbitrage - another dimension of Quant modeling as compared to derivatives modeling for market making purposes. Parts of the materials presented in this part can be used as a review on the fundamentals of the interest rate market, which would be beneficial to novice readers.

Significant amount of the materials included in this book are the original research of the authors'. Some have been published by the authors as working papers. Some have been presented by the authors in conferences, particularly, various RISK conferences, such as RISK USA, Math Week, and Credit Risk Summit USA.

To provide convenience to the reader, we have provided various

Web links for downloading some of the papers and software, etc., referenced in this book.

If there are future editions of this book, we will provide more numerical examples and possibly Excel spread sheets illustrating the methodologies presented in this book. The reader is encouraged to submit to the authors relevant numerical examples and questions and answers for possible inclusion in the future editions of the book. All included submissions will be acknowledged in the future editions of the book.

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The authors would like to thank Peter Carr of Bloomberg and Peter Ritchken of Case Western Reserve University for discussions and writing the book cover reviews.

The authors are grateful to Jeffrey Lange and Andrew Lawrence from Longitude for contributions and support of the development of the nonexploding bushy tree (NBT) technique. The authors would like to thank Hong Amy Yang of Bloomberg and Gang Liu of Citi Group for various contributions to the book, and Eugene Guillemette of Bloomberg for proof-reading the entire book and providing helpful comments.

The authors would also like to thank the following people for beneficial communications and interactions (in alphabetical order in the affiliation and then in the last name): Leif Andersen and Michael Sotiropolis from Banc of America; Anlong Li from Barclays Capital; David Bai, Rupert Cox, Thomas Fox, Man-Wing Li, Gregory Pelts, Dmitry Pugachevsky, Jin Qian, Asli Rustemli, Wei-Tong Shu, Zhenyu Yuan, Harry Zhang, Lizeng Zhang, and Robert Zvan from Bear Stearns; Patrick Hagan from Bloomberg; Vladimir Finkelstein from Citadel; L. Sankarasubramanian and Frank Xia from Citi Group; Mark Broadie, Sidney Browne, and Emanuel Derman from Columbia University; Dino Buturovic, William Kearstead, Alfredo Pastro, Klaus Toft, and Paul Young from Goldman Sachs; Lane Hughston from King's College; Eduardo Canabarro, Kanglin (Kenneth) Huang, Fei Zhou, and Qun Zuo from Lehman Brothers; Fong Liu, and Jane Yu from Merrill Lynch; Rui Kan, Jacob Xia, and Guang Yang from Morgan Stanley; David Cai from New York University; Jingzhi (Jay) Huang from Pennsylvania State University; Riccardo Rebonato form Royal Bank of Scotland; Darrel Duffie and

Ming Huang from Stanford University; Francis Longstaff, Pedro Santa-Clara, and Didier Sornnette from UCLA; Stuart Turnbull from University of Houston; and Susanta Basu and Judith Yang formerly from Goldman Sachs. The authors have presented at various RISK conferences and were also benefited from discussions with the conference attendees.

The authors are also grateful to Cheong Chean Chian, Juliet Lee, Kim Tan, Yubing Zhai, and others from World Scientific Publishing Co. for their help, encouragement, and patience in the publication process of the book.

The authors would also like to thank Vasuki Balasubramaniam and Laurie Donaldson from Risk Books and Journals for granting the reprint permission of the paper entitled "A Nonexploding Bushy Technique and Its Application to the Multifactor Interest Market Model" (Tang and Lange, 2001) published in the Journal of Computational Finance (Vol. 4, No. 4, pp 5-31). This paper is included in Section 7.9 of this book with more details added.

Yi Tang would also like to thank the following people: Rubin Braunstein, Maha Ashour-Abdala, and Roberto Peccei from UCLA; Dietrich Belitz from University of Oregon; as well as Chuni Gosh, Minwen Hong, Xiao-Dong Su, Pai (Patrick) Tang, Changjiamg Wu, Minlong Xu, Hanxiang Yu, and Tao Zhan.

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