

## THE SIMULATION OF THE ISING MODEL ON THE CREUTZ CELLULAR AUTOMATON

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This review covers mainly the results of the computer experiments for the simulation of the Ising model on the Creutz cellular automaton. Simulations have concentrated near the critical regions of the lattices, the most fruitful region for extracting information. All the dimensionalities  $2 \leq d \leq 8$  have been considered. The next-nearest neighbor and four-spin interactions have been also included in  $d = 2$  dimensions. The critical temperature, the static and dynamical critical exponents of the infinite lattice have been computed. For the analysis, the method of finite-size scaling, and the method of approximating the infinite lattice by a finite-size lattice in a certain temperature interval below the critical temperature of its magnetic susceptibility are used. The features of the Creutz cellular automaton, that can be observed only in computer experiments, are given. All the critical quantities computed, namely, the critical temperatures, the static and dynamical critical exponents are in agreement with the results of the conventional Monte Carlo method, and this leads to the conclusion that the Creutz cellular automaton can be used as an alternative research tool for Ising model investigations.

### 1. Introduction

Traditionally the Ising model is simulated by the Monte Carlo method,<sup>1</sup> by using either Metropolis<sup>1</sup> or Glauber dynamics.<sup>2</sup> They assume the spin system as a canonical ensemble, so the temperature of the system is known *a priori*.

In the conventional Monte Carlo method, the critical slowing down and the generation of random numbers are the two time-consuming factors. The efforts to reduce their contributions to the computation time led to the development of two types of algorithms: one of these was developed for speeding up the simulations about the critical points by reducing the critical slowing down, that is, by reducing the dynamical critical exponent as much as possible, but preserving the Monte Carlo method of simulation, by Swendsen and Wang<sup>3</sup> and Wolff.<sup>4</sup>

The algorithms of the other type avoid the generation of random numbers. They do not intend to reduce the critical slowing down. One of these<sup>5</sup> uses the microcanonical averages for computing the canonical averages. The others consider the spin system as a microcanonical ensemble, instead of the temperature the total energy of the system is used as a parameter for the simulation; the temperature is a computed quantity.

The first of these was developed by Creutz<sup>6</sup>: a traveling demon visits the spins and transfers kinetic energy to or from the spin it visited and simultaneously flips it, if possible, such that the total energy of the spin system is conserved during this process. The temperature of the system is computed from the the average kinetic energy of the system. This method was studied by Desai *et al.*,<sup>7</sup> for the two-dimensional Ising model. Because it can be programmed with integer arithmetics, computation time is saved. It requires random numbers only at the start of the simulation for determining the initial spin configuration; this is another factor which saves computation time. These time-saving factors are also true for the Creutz cellular automaton which is the subject of this review.

The second one of the microcanonical algorithms was developed by Vichniac.<sup>8</sup> It is a cellular automaton<sup>9</sup> called Q2R. The total energy of the system consists, only, of the Ising energy (internal energy). That is why there is no internal energy fluctuations. Therefore the kinetic energy and the temperature of the system cannot be computed from the simulation. It was studied for the two- and three-dimensional Ising models.<sup>10–18</sup>

Another microcanonical algorithm which may be thought of a hybrid of the microcanonical algorithms mentioned above was also developed by Creutz,<sup>19</sup> called the Creutz cellular automaton: a demon with a certain number of discrete kinetic energies is associated with each spin. It transfers kinetic energy to or from its own spin and simultaneously flips it, if possible, such that the total energy of the spin system is conserved during this process. If the demons are omitted, this algorithm reduces to the Q2R cellular automaton as a special case.

The Creutz cellular automaton was studied extensively for the two- to eight-dimensional Ising models and tested against the values of as many well-known quantities as possible.<sup>20–34</sup> This review focuses on this algorithm by Creutz. The model is described in Sec. 2, the methods of analysis used for extracting information from the data are summarized in Sec. 3, the results are discussed in Sec. 4, and a conclusion is given in Sec. 5.